Wireless Pers Commun
DOI 10.1007/s11277-010-0005-9

Economic Approaches for Cognitive Radio Networks: A Survey

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Abstract Efficient resource allocation is one of the key concerns of implementing cognitive radio networks. Game theory has been extensively used to study the strategic interactions between primary and secondary users for effective resource allocation. The concept of spectrum trading has introduced a new direction for the coexistence of primary and secondary users through economic benefits to primary users. The use of price theory and market theory from economics has played a vital role to facilitate economic models for spectrum trading. So, it is important to understand the feasibility of using economic approaches as well as to realize the technical challenges associated with them for implementation of cognitive radio networks. With this motivation, we present an extensive summary of the related work that use economic approaches such as game theory and/or price theory/market theory to model the behavior of primary and secondary users for spectrum sharing and discuss the associated issues. We also propose some open directions for future research on economic aspects of spectrum sharing in cognitive radio networks.

Keywords Cognitive radio · Resource allocation · Spectrum trading · Spectrum sharing · Game theory · Price theory · Market theory

1 Introduction

Cognitive radio networks [1,9,20] have been proposed to overcome the ineffectiveness of the traditional static spectrum assignment policy [12] and to facilitate effective use of electromagnetic spectrum by coexisting with licensed users through spectrum sharing. The licensed users

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Published online: 20 May 2010

are called primary users and the users of the cognitive radio network are called secondary users. To peacefully coexist with primary users, secondary users should have timely and accurate information about the usage of primary user spectrum. There are two different approaches for secondary users to get this information:

- Through Spectrum Sensing
  In this case, secondary users perform sensing of primary user spectrum in order to detect the vacant spectra called spectrum holes. Spectrum sensing is a crucial function for such opportunistic spectrum access.
- Exclusive Information from Primary Users
  In this case, the primary users explicitly provide information about the available spectrum to secondary users. In this model, the primary users get monetary or some other kinds of benefit by allowing the secondary users to use the spectrum.

In an opportunistic spectrum access scenario, there is no motivation for primary users to participate in the spectrum sharing process because they do not get any benefit by letting secondary users use their spectrum. In this approach, the primary users are inflexible and the overall responsibility of maintaining peaceful coexistence with primary users is on the secondary users, thus making the implementation aspect more complex and guaranteeing the performance harder. On the other hand, a resource trading based approach of spectrum sharing is that primary users can lease the spectrum to secondary users whenever and wherever they are not using the particular bands which in turn gives the primary users monetary or other benefits from secondary users.

Figure 1 shows spectrum trading and resource allocation as two different issues in cognitive radio networks. Effective resource allocation is the key to efficient spectrum sharing. Resource allocation can be in terms of frequency band, channel access time, transmission power etc. and can be between primary users and secondary users and among secondary users. Spectrum trading is the economic aspect of spectrum sharing in an incentive driven framework of coexistence of primary and secondary users. In a spectrum trading scenario, while primary users compete to sell the spectrum in order to maximize their revenue, secondary users compete to get the spectrum according to their needs at better price to maximize their satisfaction. Spectrum trading can be between primary and secondary users or can be among secondary users only.

There is a crucial need to study the competitive and cooperative strategies of users for multiplayer optimization of the resource allocation problem. Meanwhile, understanding the pricing issues and market structures for spectrum trading is not less important either for practical implementation of cognitive radio networks. Thus, our motivation for this work stems from the need to establish a framework to understand the possibilities and challenges of using economic approaches for deploying cognitive radio networks.

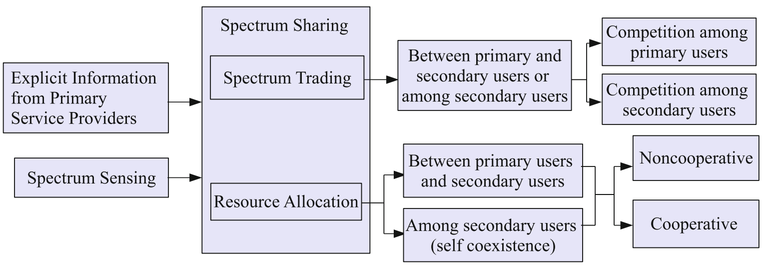

Fig. 1 Spectrum sharing issues in cognitive radio networks

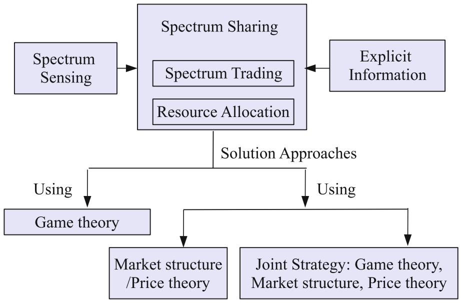

Fig. 2 Solutions for spectrum sharing in cognitive radio networks

Different approaches that have been used to model the strategic interactions for spectrum sharing are shown in Fig. 2. Game theory serves as a powerful tool in order to model the strategic behavior of primary and secondary users for their coexistence. The economic models include principles such as setting the price of the spectrum available aimed to maximize revenue of the primary users, choosing the best seller for the spectrum in order to maximize the satisfaction from the usage of the spectrum for secondary users, modeling market competition etc. Therefore, many of the existing literature on modeling the economic interactions in wireless networks use a combination of game theory, price theory and market structure.

The rest of the paper is organized as following. Different kinds of games that can be applicable to model economic interactions and decision making in cognitive radio networks and an overview of the related work are described in Sect. 2 along with the associated challenges. Price theory and market principles to model the trading activities in cognitive radio networks, related work and the corresponding research challenges are discussed in Sect. 3. An overview of the work using a combination of game theory, price theory and market theory is presented in Sect. 4. Section 5 shows the classification of the related work based on the issues and solutions. Open problems for future research are introduced in Sect. 6. Section 7 concludes the paper.

2 Game Theory

Game theory is the study of conflict and cooperation among individuals, groups or firms. It provides an analytical framework with a set of mathematical tools for the analysis of interactive decision-making processes. It is a multi-player optimization approach and the concept applies whenever the actions of several players are interdependent.

A game is formed by three fundamental components: a set of players, a set of strategies and a set of payoffs for given set of actions. A player is the one that makes decisions in the game. A strategy is a complete contingent plan, or a decision rule that defines an action that a player will select in every distinguishable state of the game. Payoff is the revenue or satisfaction of the player for a given strategy. Payoff is often expressed through utility functions. Game theory combined with market principles and price theory serves as a strong ground for modeling the economic activities of cognitive radio networks for spectrum sharing.

2.1 Cooperative and Non-cooperative Games

Games can be classified into different categories based on different criteria. A common approach is to classify games as cooperative and non-cooperative games.

2.1.1 Cooperative Game

In a cooperative game, there is no competition between players in a group and they act as a single entity to maximize the total group utility. An example is a bargaining game, which is often used to formulate the interaction among cooperative players provided that a player can influence the action of other players. In a bargaining game, the players can negotiate and bargain with each other. A general solution of the bargaining game is the Nash bargaining solution, which can ensure efficiency as well as fairness among the players.

2.1.2 Non-Cooperative Game

A non-cooperative game is the one in which players are selfish and each individual player makes decisions independently. In a non-cooperative environment, players have different (often conflicting) interests. Non-cooperative game theoretical framework is used to obtain an equilibrium solution that optimizes the payoff of all players. One of the most widely used solutions for non-cooperative games is the Nash equilibrium. Nash equilibrium is the solution at which any player in the game cannot achieve a better solution by deviating unilaterally, given the actions of the other players.

2.2 Different Game Models

Some game models that have been used extensively for analyzing the strategic interactions among users for spectrum sharing are as following.

2.2.1 Stackelberg Game

The Stackelberg leadership model [5] is a strategic game in which there is at least one player defined as the leader who can make the decision and commit the strategy on the price before other players who are defined as followers. The players engage in Stackelberg competition if one has some kind of incentive to move first. The strategy chosen by the leader can be observed by the followers, and the followers can adapt their decisions accordingly. The leader can choose a strategy such that its profit is maximized, given that the followers will choose their best responses. This solution is called the Stackelberg equilibrium.

2.2.2 Bertrand Game

In a Bertrand game [5], there are a finite number of firms that decide on the service prices simultaneously. Given the price offered by a service provider, based on a demand function, the amount of commodity requested from the users can be determined. Then, the profit is computed and used in a profit maximization problem for a service provider to obtain the best response in terms of setting the service price. For a spectrum trading scenario, the service providers are the primary users, the consumers are the secondary users and the size of the spectrum will change according to the price set by the primary users. When the service providers offer their prices simultaneously (i.e. imperfect information), Nash equilibrium is the solution. The interaction in Stackelberg game is more dynamic due to the timing in strategy adaptation compared to the Bertrand model. If the assumption of perfect information is released and all firms decide their service prices simultaneously, Stackelberg model reduces to Bertrand model.

2.2.3 *Cournot Game*

In Cournot game [5], the competition is in terms of the quantity of the commodity. The decision of each user is affected by the strategies of other users and the decisions are made simultaneously.

2.2.4 *Coalition Game*

Cooperative game theory provides analytical tools to study the behavior of rational players when they collaborate. The group of cooperating players that can strengthen the players' position in the game, is called a coalition, and all players forming a coalition act as a single entity. Based on application oriented approach, coalitional games can be classified into three categories [31]: canonical (coalitional) games, coalition formation games and coalitional graph games. In canonical games, no group of players can do worse by joining a coalition than by acting non-cooperatively. In coalition formation games, forming a coalition brings advantage to its members but the gains are limited by a cost for forming the coalition. In coalitional graph games, the coalitional game is in graph form and the interconnection between the players strongly affects the characteristics as well as the outcome of the game.

2.2.5 *Game with Learning*

In a competitive resource market, the information available may be incomplete and/or imperfect. In such cases, the players have to evolve and learn the behavior of other players from history. Such evolutionary games are often called games with learning and are closely related to the concept of "repetition" in games (described next). If the games are "repeated", players can learn and adapt their behaviors and strategies in subsequent rounds of the game.

2.2.6 *Repeated Games*

A repeated game [5] is an important tool in order to understand the concepts of reputation and punishment in game theory. A repeated game allows a strategy to be contingent on the past moves, thus allowing threats and promises about future behavior to influence current behavior, which create possibilities for cooperation among greedy users. If a greedy user behaves selfishly and chooses the strategy to optimize his/her individual payoff, it can enjoy the benefit in one round. However, if this user has to depend on others as well for future rounds of the game, it will be punished by them. Players must therefore consider the effects that their chosen strategy in any round of the game will have on opponents' strategies in subsequent rounds.

2.3 Applications of Game Theory in Spectrum Sharing

Game formulations can be used for multiplayer optimization to achieve individual optimal solution for resource allocation. The use of game theoretic models for resource allocation has mainly focused on issues such as admission control, throughput optimization, power control, channel allocation etc. Table 1 summarizes the related work on the use of game theory for resource allocation, in terms of the specific issue addressed, approach/model(s) used and the solution proposed. These works are explained next.

Table 1 Summary of related work on resource allocation using game theory

Paper	Issue(s) addressed	Approach/specific model(s) used	Solution
[6]	Power control, throughput control	Non-cooperative, distributed game	Nash equilibrium (Optimal for power, optimal or suboptimal for network throughput)
[7]	Power control, rate adaptation, subchannel assignment	Non-cooperative, distributed game	Nash equilibrium (Transmitted power)
[10]	Channel/power allocation	Non-cooperative/cooperative, distributed game	Nash equilibrium (Radio range)/ nash bargaining solution
[33]	Spectrum allocation (Channel switching)	Non-cooperative, distributed game (Modified minority game)	Nash equilibrium (Channel switching probability)
[36]	Channel access time	Cooperative Stackelberg game (between primary and secondary users), non-cooperative payment selection game (among secondary users), distributed game	Nash equilibrium (Payment vector)
[3]	Spectrum assignment	Cooperative, distributed, bargaining game	Bargaining based solution (Spectrum usage)
[19]	Channel allocation	Cooperative, distributed game	Correlated equilibrium (Spectrum access)
[8]	Packet forwarding	Cooperative, distributed game (Repeated coalitional game)	Min-max fairness, average fairness and market fairness investigated
[32]	Spectrum sensing	Cooperative, distributed game (Non-transferable utility coalitional game)	Performance compared with non-cooperative and centralized scheme

In [6], a game theoretical approach is proposed for distributed resource allocation in wireless networks. Power control at the user level and throughput control at the system level are linked through non-cooperative games.

In [7], a distributed non-cooperative game is proposed for joint subchannel assignment, adaptive modulation and power control for multi-cell multi-user OFDMA networks. In order to improve the performance of Nash equilibrium points, a virtual referee is introduced in the system that can modify the rule of the resource competition game for efficient resource sharing.

In [10], the authors modeled the channel/power allocation for cognitive radios considering IEEE 802.22 [13] framework. The strategic behavior of the system was studied considering the limit on the total interference from all opportunistic transmissions for each primary user as well as the minimum SINR requirement of the cognitive radios and a cooperative scheme based on Nash bargaining solution was proposed for optimal channel/power allocations.

In [33], the self-coexistence of multiple overlapping IEEE 802.22 networks operated by multiple wireless service providers that compete for resources and try to seek a spectrum band without any interference from other coexisting IEEE 802.22 networks, was investigated from a game theoretic perspective. The dynamic channel switching was modeled as a distributed modified minority game (MMG), in which each user has to decide whether to leave a particular band or to continue using it when another user also appears in the same band.

In [36], a cooperative cognitive radio framework is formulated as a Stackelberg game where primary user acting as the leader, selects some of the secondary users to be the cooperative relay, and in return, leases portion of the channel access time to them for their own data transmission. Selected secondary transmitters, acting as the followers, can use the wireless channel only if they cooperate with the primary link and meanwhile make a certain amount of payment to the primary system.

In [3], the authors propose a local bargaining approach to achieve distributed conflict free spectrum assignment adapted to network topology changes assuming that there is a collaboration between network nodes to improve system utility. In this paper, the authors propose Fairness Bargaining with Feed Poverty to improve fairness in spectrum assignment and have derived a lower bound on the spectrum assignment (poverty line) that each node can get from bargaining.

In [19], a decentralized dynamic spectrum access scheme is proposed for cognitive radios considering the application domain as a set of collision channels from game theoretical perspective. The authors proposed the use of an adaptive procedure called Regret Tracking, which converges even when multiple users are adapting their behavior simultaneously, for which correlated equilibrium (in terms of channel allocation) is investigated.

In [8], an approach based on coalition games is proposed for symbiotic cooperation between boundary nodes and backbone nodes in selfish packet forwarding wireless networks. Different fairness criteria are investigated including market fairness. In addition, a joint protocol is designed using both repeated games and coalition games for packet-forwarding and it has been shown that the network connectivity can be significantly improved using the proposed protocol compared to using pure repeated game approach.

In [32], a distributed collaborative spectrum sensing algorithm is developed based on a dynamic coalition formation game among secondary users to improve the overall probability of miss detection with increase in the probability of false alarm as the cost for coalition formation.

2.4 Research Challenges of Using Game Theory

Although, game theory is a powerful tool to model and analyze the interactions among primary and/or secondary users for spectrum sharing, how realistic are the assumptions in the models and how close the models are to the possible implementation of cognitive radio networks, are yet open issues. These issues are challenging especially because of the need of dynamic access of the spectrum and heterogeneous requirements on the quality/quantity of the offered spectrum for different users in the network. Some of such challenges for specific game models, are explained next.

The availability of perfect knowledge in Stackelberg model may be quite costly for distributed implementation of cognitive radio networks. How to efficiently manage the information flow among leaders and followers in a dynamic scenario is a big challenge. Even if the information can be made available to all players, the overhead due to this information exchange (which increases with the number of players in the game) can not be ignored. Although, the prices and profits of some service providers may be higher at Stackelberg equilibrium than at Nash equilibrium [25], Bertrand/Cournot game models (thus Nash equilibrium solution) may incur relatively less overhead for the distributed approach because of the simultaneity of moves.

Cooperative framework is often used to model the spectrum sharing scenario in cognitive radio networks. However, finding users with common interest to form coalitions and get them

to act cooperatively, itself is a big issue. Even if there exist users that can be symbiotic, the change in network topology, change in channel conditions, motivation towards cooperating with other users etc. may cause the coalition not to be stable. Introducing incentive or monetary gain based schemes can be quite expensive and inefficient due to divergence of user interests.

The concept of repeated games provides a good direction towards counteracting the possibility of collusive behavior in a network by certain selfish players and to adapt one's strategies accordingly. However, algorithms are necessary to effectively estimate long-term profits. In a practical network, the overhead to maintain the database of the strategies of each player and to update it is a huge challenge in a distributed scenario. In addition, when the game is being played the second last time or the last time, the future profits are not meaningful. So, in such a case, the concept of repeated games may not be effective enough. Besides, if an individual deviates from the optimal strategy for the system for its own benefit, to find out which user it was, is another issue. It may be possible to locate the group from which the deviating action occurred but locating the exact player from the group may still be difficult considering the time limitations, especially when users have the right not to disclose their strategies.

3 Price Theory and Market Theory

Price theory explains how relative prices are determined and how prices function to coordinate the economic activities. For incentive/monetary gain based spectrum sharing, appropriate pricing schemes are necessary for setting up the price of the spectrum, formulating economic models and maximizing the payoffs of both primary and secondary users. Pricing is an important issue not only to maximize the revenue of the service providers but also to prevent unnecessary competition (to reserve the resources) and to allocate the radio resource efficiently.

3.1 Price Theory

Auction and bargaining are the popular pricing schemes for resource trading. These are explained next.

3.1.1 Auction Theory

An auction [16] is a decentralized form of trading, widely known for providing efficient allocation of scarce resources. Sellers use auctions to improve revenue by dynamically pricing based on buyer demands. Buyers benefit since auctions assign resources to buyers who value them the most. In a game-theoretic auction model, the action set of each player is a set of bid functions or reservation prices. Each bid function maps the player's value (in case of a buyer) or cost (in case of a seller) to a bid price.

There are different kinds of auctions such as English auction, sealed first price auction, Vickrey auction, double auction etc. English auction is the ascending price auction in which bidders bid openly against one another, with each subsequent bid higher than the previous bid and the highest bidder gets the commodity at his/her bid. In sealed first-price auction, all bidders simultaneously submit sealed bids so that no bidder knows the bid of any other participant. The highest bidder pays the price he/she submitted. Vickrey auction is the sealed bid second-price auction, in which the bidders submit sealed bids and the highest bidder

wins, but pays only as much as the second-highest bid. Vickrey-Clarke-Groves auction [16] is a generalization of Vickrey auction for multiple items. When there are multiple items and multiple buyers, double auction is often used to model the double competition.

3.1.2 Bargain Theory

Bargaining is a type of negotiation in which the buyer and the seller of a commodity or service dispute the price that will be paid and the exact nature of the transaction that will take place, and eventually come to an agreement. Bargaining is an alternative pricing strategy to fixed prices. In a bargaining scenario, the buyer's willingness to pay is dominant over the actual price of the commodity.

3.2 Market Theory

A market is the most efficient known mechanism for the allocation of goods and services. A market consists of sellers and buyers of a commodity or service. Appropriate pricing schemes are necessary to maintain the stability of the market. Some of the useful concepts of market theory used for spectrum trading are as following.

3.2.1 Monopoly, Oligopoly and Competitive Equilibrium

Monopoly is the simplest market structure when there is only one seller in the system. Since there is a single seller in this market structure, the seller can optimize the trading to achieve the highest profit based on the demand from buyers. In an oligopoly market structure, a small number of firms dominate the market. The firms compete with each other independently to achieve the highest profit by controlling the quantity or the price of the supplied commodity. Oligopoly differs from monopoly in the sense that there are a multiple (few) firms providing the same service, thus making it necessary for each firm to take into account the strategies of all other firms. In a monopolistic context, the pricing is a single level of game between users (buyers). However, when there are multiple sellers in the market for the same commodity, the competition between them can highly affect the results of price determination. This competition introduces an additional level of game among the service providers. Buyers demand less as the price of a commodity increases. On the other hand, sellers tend to produce more as the price increases. The price at which the quantity supplied of a product/service and the quantity of it demanded are equal, is called the market equilibrium price and when the environment is competitive with flexible prices and many traders, the equilibrium is called competitive equilibrium.

3.3 Applications of Price Theory and Market Theory in Spectrum Sharing

In this section, an overview of the existing work using price theory and market principles to model economic interactions for spectrum sharing is presented (summarized in Table 2 in terms of the specific issue addressed, structure, approach/model(s) used and the solution proposed). Although price theory and market theory provide us models to address pricing issues and market stability, to model the interdependency of the sellers and buyers in the market and their strategic interactions, game theory is used. So, many literature use a combination of price theory, market theory and game theory for analyzing spectrum sharing. An overview of these works is presented in Sect. 4.

Table 2 Summary of related work on spectrum sharing using price theory/market theory

Paper	Issue(s) addressed	Structure	Approach/specific model(s) used	Solution
[30]	Spectrum allocation	Cooperative, centralized	Optimization (Multi-unit Vickrey auction)	Optimal price
[4]	Spectrum access	Cooperative, centralized	Optimization	Optimal price
[21]	Spectrum trading	Competitive (among secondary users), Distributed	Joint pricing and marketing theory	Pricing solution obtained through market equilibrium and disequilibrium

In [30], the problem of a CDMA operator participating in a dynamic spectrum allocation scheme is addressed in a cooperative framework based on multi-unit Vickrey auction. A spectrum manager implements DSA by periodically auctioning short-term spectrum licenses and a pricing driven solution based on the willingness to pay of each user is introduced.

In [4], a framework based on an auction mechanism was presented for dynamic spectrum access using classical optimization approach. In the system model considered in [4], multiple spectrum buyers submit spectrum demand function, which is based on piecewise linear price demand (PLPD), to the spectrum owner that formulates an optimization problem to maximize revenue under an interference constraint. The authors propose to restrict the interference constraints and reduce them into a number that grows linearly with the number of buyers.

In [21], a spectrum trading model based on multiple markets for different frequency bands is proposed between the primary and secondary services. The authors have investigated two different cases: the first one with equilibrium pricing where spectrum supply is equal to spectrum demand and the second one is the case where the sellers do not offer the equilibrium price, and have proposed models for both cases using linear feedback time-invariant control systems. Classical control system stability techniques are used to analyze the dynamics of market behavior under both cases.

3.4 Research Challenges of Using Price Theory and Market Theory

Though price theory and market theory can be applicable to model the spectrum trading for cognitive radio networks, the information exchange required for pricing and negotiation is a big challenge. Eg. Vickrey-Clarke-Groves auction is one of the most used auctions for resource trading. It can be used to achieve a socially optimal allocation. However, it requires gathering global information from the users, which is a huge challenge for distributed implementation of cognitive radio networks especially when the available information is incomplete. The communication overhead and computational complexity to gather and manage global information in a distributed scenario may be quite costly.

To perform bargaining, users form groups and bargain with other groups. However, the larger the groups are, the more is the complexity of bargaining due to high costs of synchronization and communication overhead. So, efficient formation of bargaining groups and effective communication between them in a distributed spectrum sharing scenario is also another issue. In addition, the stability of the bargaining groups formed in a network with rapidly changing topology and other underlying conditions is another issue. To dynamically

split and merge to firm optimal coalitions in a network with only local information available, is also a huge challenge.

The pricing theory with per unit price of the resource same for all users for any amount of resource demanded may not produce highest revenue for the sellers. So, a discriminatory pricing scheme as proposed in [4] may be better in terms of maximizing the revenue. However, the computational complexity of this kind of scheme is yet a big issue.

4 Joint Strategy: Game Theory, Market Theory and Price Theory

While market principles and price theory are needed to model economic activities, game theory is necessary to analyze the interdependency and strategies of the users for spectrum sharing. So, many literature use a combination of these to investigate spectrum trading and resource allocation in cognitive radio networks. Table 3 summarizes the related work using joint strategy in terms of the specific issue addressed, structure, approach/model(s) used and the solution proposed, each of which is briefly explained next.

Table 3 Summary of related work on spectrum sharing using joint strategy

Paper	Issue(s) addressed	Structure	Specific model(s) used	Solution
[2]	Power control	Non-cooperative, distributed	Non-cooperative game, pricing mechanism	Nash equilibrium (For uniformly strictly convex pricing function)
[11]	Channel access	Non-cooperative, distributed	Non-cooperative game, SINR auction, power auction	Nash equilibrium (Bidding profile, power profile)
[34]	Joint power/ channel allocation	Non-cooperative/cooperative (among CR pairs), distributed	Non-cooperative game, pricing based cooperation	Nash equilibrium/ Pareto optimum boundary (power vector)
[23]	Spectrum trading	Competitive (among primary services), centralized/distributed	Non-cooperative game, Bertrand game, repeated game, Oligopoly market	Nash equilibrium, optimal spectrum price
[22,24]	Spectrum trading	Competitive (among secondary users), centralized/distributed	Non-cooperative game, Cournot game, Oligopoly market	Nash equilibrium (Spectrum size)
[25]	Spectrum pricing	Competitive (among service providers), centralized	Non-cooperative game, Stackelberg game, Bertrand game, oligopoly market	Stackelberg equilibrium, Nash equilibrium (Spectrum price)
[26]	Spectrum trading	Market equilibrium/ competitive/cooperative (Among primary service providers), distributed	Non-cooperative game, optimization, oligopoly market	Market equilibrium, Nash equilibrium, optimal price
[28]	Spectrum trading	Competitive (among primary users, among secondary users), distributed	Non-cooperative game, (among primary users), evolutionary game (among secondary users), Oligopoly market	Nash equilibrium (Spectrum price), evolutionary equilibrium (Spectrum size)

Table 3 Continued

Paper	Issue(s) addressed	Structure	Specific model(s) used	Solution
[15]	Spectrum allocation	Non-cooperative, distributed	Non-cooperative game, double auction, pricing based allocation	Nash bargaining solution (lower bound on payoff), competitive equilibrium (without user collusion)
[17]	Slotted resource allocation	Competitive (among providers), distributed	Non-cooperative two stage pricing game	Nash equilibrium of the pricing game (under mild conditions)
[35]	Spectrum trading/access	Competitive (among primary users), centralized/distributed	Non-cooperative game	Nash equilibrium (spectrum price)
[14]	Spectrum leasing	Non-cooperative (between primary and secondary users)	Non-cooperative power control game	Nash equilibrium (Transmission power)
[29]	Joint spectrum bidding and pricing	Non-cooperative, centralized	Non-cooperative game, sealed-bid double auction	Nash equilibrium (Spectrum price)
[27]	Spectrum trading	Non-cooperative (among TV broadcasters), (among WRAN users), Distributed	Non-cooperative game, generalized fading memory scheme, microeconomic approach	Market equilibrium (Spectrum price)

In [2], the CDMA uplink power control in a multicell CDMA wireless network model is addressed as a non-cooperative game. The game incorporates a pricing mechanism that limits the overall interference and preserves battery energy of mobiles. The concept of outage probability was introduced as a performance metric for the quality of the channel. Distributed iterative power algorithms are analyzed using an outage probability based utility function for a generalized fading channel model.

In [11], a non-cooperative game was formulated to address the problem of spectrum sharing among users using spread spectrum signaling, in a distributed scenario to access the channel subject to a constraint on the interference temperature at a measurement point and two auction mechanisms: : SINR auction and power auction, are proposed for allocating received power.

In [34], a joint power/channel allocation scheme is proposed using a distributed pricing approach for cognitive radio networks and a frequency dependent power mask constraint is introduced for secondary users in addition to maximum transmission power constraint and minimum SINR constraint.

In [23], the issue of spectrum pricing in cognitive radio network is addressed for multiple primary services and 1 secondary service. The trading of spectrum between primary and secondary services was modeled as an oligopoly market. A Bertrand game model was applied for price competition among primary services to obtain the Nash equilibrium pricing. Distributed algorithms were presented to obtain the solution of the dynamic game, when primary services have to make decisions based only on the spectrum demand from the secondary

service. A repeated game was formulated to analyze the behavior of selfish primary operators that try to deviate from the equilibrium point to increase their profit at the cost of lower profit to other primary operators.

In [22,24] the competition among multiple secondary users for spectrum offered by 1 primary user was modeled using Cournot game. In [24], the problem is formulated as an oligopoly market competition and the spectrum allocation for secondary users is obtained through non-cooperative game. The competition is in terms of the size of the spectrum they request. A dynamic game is formulated in which the selection of strategy by the secondary users is solely based on the pricing information obtained from the primary user.

In [25], the authors modeled service competition and pricing in a WiMAX and WiFi based heterogeneous wireless access network using non-cooperative Stackelberg and Bertrand game models respectively and the performance was compared for the two models.

In [26], spectrum trading for cognitive radio networks was investigated considering multiple primary services that are willing to sell the available spectrum to the secondary service. Distributed algorithms were presented for three different pricing schemes: market equilibrium, competitive and cooperative pricing models and the performance of all three schemes were compared.

In [28], the problem of spectrum trading with multiple primary users selling spectrum opportunities to multiple secondary users is considered. The competition among primary users is formulated as a non-cooperative game where each primary user sets the size of spectrum to be shared and the price of the spectrum such that its own payoff is maximized. It is assumed that the secondary users can evolve over time to buy the spectrum opportunities that provide the best payoff in terms of performance and price.

In [15], spectrum allocation among primary and secondary users is modeled as a bilateral pricing process to maximize the utilities of both primary and secondary users and a distributed collusion-resistant dynamic pricing approach with optimal reserve prices was proposed to achieve efficient spectrum allocation while combating user collusion. Double auction scenario was considered for the pricing game. A belief function was introduced that builds up certain belief of other players' future possible strategies for each user to assist its decision making.

In [17], the competition among providers was studied on a non-cooperative game theoretic framework. The authors introduced a pricing model dealing with how for fixed prices, total demand is split among providers following Wardrop's principle and determined the existence and uniqueness of Nash equilibrium under mild conditions.

In [35], the economic interactions are modeled considering both price of the offered spectrum and its quality. The analysis scenario consists of multiple self interested spectrum providers operating with different technologies and offering the spectrum at different costs that compete with each other to get potential customers that are grouped into two categories: quality sensitive group and price sensitive group. In [35], the authors have proposed a practical price updating strategy using structured stochastic learning for which the price is shown to converge to the optimal equilibrium.

In [14], a game theoretic framework was developed to facilitate dynamic spectrum leasing (DSL) in cognitive radio networks in which primary users are also included as active decision makers in a non-cooperative game with secondary users by selecting an interference cap on the total interference they are willing to tolerate.

In [29], the joint spectrum bidding and pricing scheme was proposed for dynamic spectrum access in the exclusive usage model for IEEE 802.22 based cognitive radio network. Multiple TV broadcasters offer the available TV bands and WRAN service providers bid for these TV bands. A sealed bid double auction scenario was considered for the procurement

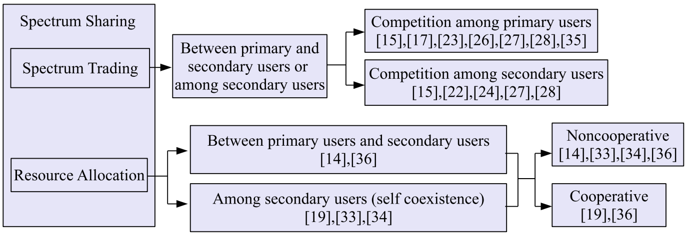

Fig. 3 Classification of related work on spectrum sharing based on resource allocation and spectrum trading

of TV bands from TV broadcasters in terms of the number of TV bands and the trading price. After buying TV bands, multiple WRAN service providers compete with each other to sell the spectrum to WRAN users. A non-cooperative game was formulated to model the competitive environment for bidding and pricing strategies.

In [27] the authors have proposed a market-equilibrium based model for spectrum trading between primary and secondary services using supply and demand functions. A non-cooperative game is formulated between primary and secondary users where a distributed generalized fading memory algorithm is used by the secondary service to estimate spectrum price and adjust spectrum demand accordingly so that the market equilibrium can be reached for the price and size of the spectrum allocated for the secondary service by the primary service.

5 Classification of Related Work Based on Issues and Solutions

In this section, we categorize the related work on modeling the economic interactions in cognitive radio networks on three different bases as following.

Figure 3 shows the classification based on different aspects of spectrum trading and resource allocation as described in Sect. 1. The related work on spectrum trading between primary and secondary users address competition among primary users and competition among secondary users. While [17,23,26,35] deal with price competition among primary users/service providers for profit maximization, [22,24] address the competition among secondary users in terms of the size or quality of spectrum demanded. On the other hand [15,27,28] address the competition among primary services as well as among secondary users for spectrum trading.

In terms of resource allocation, while [14,36] consider the coexistence between primary service providers and secondary users, [19,33,34] explore self coexistence among secondary users. Although both [14,36] consider the coexistence between primary and secondary systems, [36] is based on the cooperation between primary and secondary users, while in [14], the framework is non-cooperative. The payment selection game among secondary users in [36] however is a non-cooperative game. For self coexistence among secondary users, while [33] proposes a non-cooperative dynamic channel switching game, [19] considers a cooperative approach for channel allocation. On the other hand, [34] considers a non-cooperative framework and a pricing based cooperative approach for self coexistence of secondary users.

Figure 4a shows the classification based on the particular game models described in Sect. 2. Both [25,36] use Stackelberg game model. However, while the approach considered in [36] is a cooperative framework between primary and secondary users, [25] uses

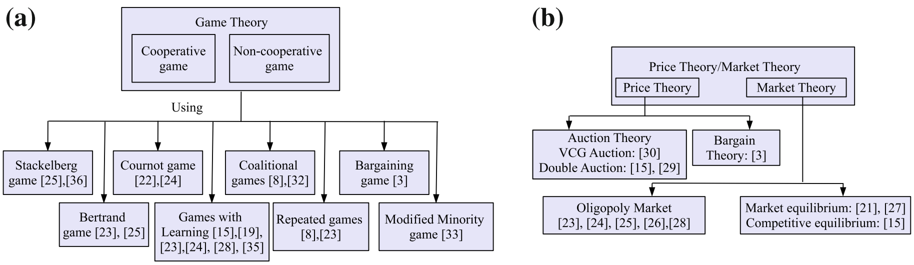

Fig. 4 Classification of related work on spectrum sharing based on solution approaches

Stackelberg game for price competition among WiMAX and WiFi service providers. However, when all players in the game should act simultaneously, the price competition game can be modeled using Bertrand game as in [23,25]. When the competition is not in terms of price, but in terms of the size of the spectrum, Cournot game was applied to model the competition among secondary users [22,24].

In a distributed scenario with incomplete information, the players have access to local information only and they have to learn about the strategy of other players adaptively. Such interactions can be modeled using games with learning [15,19,23,24,28,35]. In [23], when a primary service provider has the information about the demand from secondary service but no information about the current pricing strategy of other primary services, a non-cooperative game with learning was used to decide its strategy based on the past strategy of other primary services. Similar approach is applied but in case of secondary users in [24]. [28] addresses the issue for both primary and secondary users when there are multiple primary services and multiple secondary users. In [15], belief assisted approach (using belief function) was used for selfish users to reduce pricing overhead. In [35], a price updating strategy was proposed using structured stochastic learning when the sellers have no knowledge about each others' strategy and also about the consumer population. In [19], the authors have used a learning based game called Regret Tracking for channel allocation among cognitive radio users.

Coalitional game was used in [32] for collaborative spectrum sensing. In [8], coalition game combined with repeated game was used to make the boundary nodes and backbone nodes collaborate for packet forwarding. In [23], repeated games were used to prevent selfish primary services from deviating unilaterally for individual profit that may lower the profit for other primary services. The combination of repeated and coalitional games can be a very effective way to introduce cooperation among network users. A bargaining game was used in [3] in a cooperative and distributed framework for spectrum assignment. An MMG was used in [33] in a non-cooperative framework for channel switching.

Figure 4(b) depicts the classification based on the particular models/approaches from price theory and market structure described in section 3. In [30], VCG auction was used for dynamic spectrum allocation in a cooperative and centralized framework. Double auction scenario was considered in [15,29] in a non-cooperative frame- work. Bargaining game was used in [3] in a cooperative and distributed framework for spectrum assignment. Oligopoly market structure was considered for analyzing pricing schemes in [23–26,28]. While [21,27] considered market equilibrium as the pricing solution, [15] used competitive equilibrium for the case without user collusion in the network.

The applicability of particular game models or pricing schemes/market structure depends on many factors. The selfish behavior of network users and the limited amount of information available usually requires distributed schemes for spectrum sharing. On the other hand, the synchronization issues and the information exchange overhead in a distributed system is a

huge research challenge. The implementation of pricing schemes can also cause significant communication overhead to the system. In many cases, Nash equilibriums are not efficient and Pareto optimal solutions provide higher revenues/payoffs. However, the cost to make the users act cooperatively in a distributed environment may be quite high, which leaves users no other choice but Nash equilibrium. The trade off is in terms of information exchange and computation complexity versus distributedness.

6 Open Research Problems

Some possible directions for future research in investigating the economic interactions for spectrum sharing in cognitive radio networks are as following.

6.1 Coalition Formation and Communication Overhead

Many of the literature that propose cooperative strategies for primary/secondary users have not considered the cost for cooperation. The cost can be the power required for negotiation, delay because of the information exchange etc. In a practical scenario, it is not reasonable to neglect this cost for spectrum management especially in a resource constrained network like cognitive radio. Therefore, investigating the overhead caused by the communication to form a coalition for cooperation for spectrum sharing can be an interesting direction for future research.

6.2 Bidder Collusion

While the objective of primary services to participate in spectrum trading is to maximize their revenue, the purpose of secondary services is to get the spectrum as cheap as possible such that the required quality of service is also maintained. So, it is likely that the bidders may attempt to lower the price of the spectrum offered by the primary services by acting collusively. Bidder collusion [18] is probably the most serious practical threat to the revenue of the primary users but there has not been much work on this. So, another possible direction for future research may be to study the interactions in presence of bidders' collusion and to explore its effect on the payoff of primary and secondary users.

6.3 Incentive Driven Spectrum Sensing

Spectrum sensing, a crucial function for opportunistic spectrum access, requires significant share of the network resources (energy, time etc.). So, selfish secondary users tend to take a free ride as far as possible by overhearing the sensing results from other secondary users. To prevent such behavior and to maintain the motivation to perform sensing, incentive or reward driven sensing schemes can be another possible direction for further research.

6.4 Trust and Security in Collaborative Sensing

Trust and security is one of the key issues in cognitive radio networks. The presence of malicious sensors and their possible attack against honest sensors can pose a serious threat to the reliability of the results obtained from collaborative spectrum sensing. Investigating

different issues such as how can malicious sensors attack the honest sensors, how to detect them, how to counteract such attacks, the criteria that the reliability of the results obtained from collaborative sensing can be maintained despite the presence of malicious sensors etc. using game theory can be a very interesting research area.

6.5 Assumption of Rationality and Complete Information

Although game theory is a powerful tool for modeling resource allocation problems, it inherently assumes that the players are rational and often the payoff functions of all players are assumed to be common knowledge, which in many scenarios may not be valid. Game theory merely suggests what strategies should be taken to maximize individual/total utility. It does not provide much insight into the analysis of what the players are most likely to do and how much rational the players are likely to be when the information available is incomplete or faulty. Therefore, further study is needed to investigate the factors that may make the players irrational and to predict the natural behavior of players in addition to the strategies that are towards maximizing the individual or total utility.

7 Conclusion

We have described different types and models of games, price theory and market principles that have been used to model the economic activities of primary and secondary users for resource allocation and spectrum trading. An extensive summary of the related work on economic approaches has been presented with the classification based on the spectrum sharing issues and solutions. We also discussed the research challenges of using game theory and price theory/market theory for their application in cognitive radio research. We discussed the open research problems and proposed some interesting directions for future research about economic approaches in cognitive radio networks.

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