Jouni Laitinen

GAME THEORY IN WIRELESS NETWORKS

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Asiasanat: asiasana1, asiasana2, asiasana3, …

ABSTRACT

Laitinen Jouni

Game Theory in Wireless Networks – A Literary Review

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Text of the abstract is written here. Make sure the abstract fits in this page with the keywords as well.

Keywords: keyword1, keyword2, keyword3, …

GRAPHS

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1. Introduction

Some general information about why you should read this thesis, where you could find game theory.

introon esimerkki ja miten verkot ja peliteoria yhdistetään, miksi hyödyllistä. Esitellään myös tutkimusongelmat

A brief overview of game theory

Merriam-Webster defines game theory, as “the analysis of a situation involving conflicting interests (as in business or military strategy) in terms of gains and losses among opposing players”. What this means is that game theory is used to describe decision making situations and the interaction that happens. For this, the mathematical tools that game theory provides are a valuable asset. Due to the fact that the tools that game theory provides are quite abstract, they can be used in various situations ranging from the strictly mathematical to social sciences and information technology. In recent years there has even been some advances in to using game theory to analyze human decision making in a field called Neuroeconomics (Glimcher, 2003; Camerer, Loewenstein and Prelec, 2005). The next chapter will give a more detailed description in to the history of Game Theory.

1.1 History

Game theory has a very long history and the first results of game theoretic modeling can be seen in the Talmud(Aumann & Maschler, 1985) where the results of bankruptcy are considered. Augustin Cournot (1801 – 1877) formed a model of oligopoly(Cournot, 1838) which models the interaction between a small number of sellers. Currently, the field of game theory is considered to have gotten formed in 1944 by the publication of “Theory of games and economic behavior” by John von Neumann and Oskar Morgenster. The book has essential tools to solve games such as backwards induction. As for games the book features zero-sum games, non-zero sum games as well as games with perfect and imperfect information. Most of these are covered later in this thesis.

Next big step froward was when developed by John Forbes Nash. Nash's doctoral thesis included the definition for the Nash Equilibrium. The thesis served as a foundation for four articles that include non-cooperative games(Nash, 1951) and bargaining (1950). For these developments Nash was awarded the Nobel Memorial Price in Economic Sciences in 1994. The price was awarded together with Reinhard Selten and John Harsanyi. All together the Nobel Price has been awarded to eight game theorists. In in recent years game theory has been used as a base of mechanism design (Fudenberg & Tirole, 1991). This field is sometimes called reverse game theory.

1.2 What is a strategic game?

In a game strategic there are different actors who have to make decisions on how to act upon certain rules and have certain preferences over out comes. This means for example that a situation when two friends are deciding on where to eat and they both have a favorite restaurant can be modeled as a strategic game. The definition of such a strategic game is as follows (Osborne, 2004)

- a set of players
- each player has a set of actions
- each player has some preferences over the possible outcomes

In this the players are the two friends. Both of them can choose where they will eat but would prefer to go each own favorite restaurant. The game derived from these premises is usually called the “battle of the sexes” or “Bach or Stravisky”.

- Players: the two friends
- Set of actions: each player has the choice between {restaunt1, restaurant2}
- Outcomes: each one prefers to eat at her favorite restaurant

We will analyze this game later.

1.3 Different types of games

There are many ways one could categorize different types of games. Avinash Dixit and Susan Skeath gave a very good list of different types of games in their

book “Games of Strategy”. In the book they divide games in to six different categories(Dixi & Skeath, 2004, 20-27)

- Do the players take turns? If they do the game is called a sequential game and the players make their decision simultaneously the game is called a simultaneous games.
- Zero sum games, that is are the players interests opposite or do they share some interests.
- Repeated games. Is the game going to be played more than once. If played only once then the game is called a one-shot game. If played more than once then the game is called a repeated game.
- Games of perfect information. Do the players know everything about their opponents or is there uncertainty in the game? If there is uncertainty in the game then it is called a game of imperfect information. If one player know more about the game than her opponent(s) then the game is called a game with incomplete, or asymmetric, information.
- Can the rules be manipulated? If the rules can be manipulated then the pregame becomes the real game as this will surely effect the outcome of the game.
- Can cooperation agreements be enforced? If agreements indeed can be enforced then the game is called a cooperative game. If not then the game is called a non-cooperative game.

The games that are going to be looked at can be divided in to two main categories: Cooperative games and non-cooperative games. The reason for this is quite simple because the reasoning and logic behind cooperative game is different than the reasoning behind non-cooperative. In cooperative games the players are trying to maximize their outcomes by working as a group in an environment where agreements can be enforced. This is quite the opposite in non-cooperative games where agreements can not be enforced and therefore it is in every players interest to maximize her payoff. One other reason for dividing them in to these two categories is that both cooperative and non-cooperative games can have games with the other properties listed above within the games themselves. This is to say that there can be a repeated non-cooperative game with imperfect information. That is also going to be one of the examples that is going to be discussed in the section of non-cooperative games in wireless networks.

1.3.1 Cooperative Games

Games of cooperation are games where agreements enforceable (Dixit & Skeath 2004). This means that the players decisions are made in a group and that all

members of the group will act according to the decision or a game where all players act according the agreements that can be forced collectively or directly. By doing so the players cooperate to maximize their payoffs. Working together they can for example form coalitions that aim to increase the payoff of the coalition and thus increase the payoff to each member.

1.3.2 Non-cooperative Games

Non-cooperative games are games where agreements can not be forced and thus the main focus of each player is to maximize their own payoffs. While this does make it seem like every player always acts selfish it is not always the case. In a later chapter there will be an example of a game where cooperation still guarantees the best outcome. Probably the most famous non-cooperative game is called the prisoners dilemma. The ideas behind prisoners dilemma is quite simple. Two friends have committed a crime and been detained by the police. The police have enough information on the two to convict them because of a smaller crime that two did earlier. They also know that the two did a larger crime but they lack the information to prosecute. The police make an offer where the person who rats out the accomplice gets to walk free but the other person gets a longer sentence. If both of them talk then both of them will serve a long sentence. Both prefer walking free to small sentence to a long sentence. This is also how her partner in crime thinks. Therefore, this situation can be formed as the following game

Payers: the two criminals.
Strategies: {talk, don't talk}
Preferences: walk free > short sentence > medium sentence > long sentence

The next chapter provides everything needed in order to analyze and solve this situation.

1.4 How are games solved?

In this chapter covers how different types of games can be analyzed and solved. Due to the fact that this chapter is going to be notation heavy there will be quite a few examples. For more advanced readers should look at (Osborne, 2004) as it contains more formal definitions of the following games. For a reader wanting more explanations (Dixit & Skeath, 2004) might be good because it

contains more thorough explanations and doesn't have as much formal equations as former.

Game theory assumes that the players are rational and acts rationally to maximize their payoffs (Osborne, 2004). What this means is that the player can assign some kind of values to different outcomes of the games and act rationally to maximize it. Players are assumed to have some common knowledge. For more thorough commentary on what rationality implies, (Osborne, 2004, 6-7; Dixit & Skeath, 2004, 29 - 32) is a good place to start. For criticism on the rationality assumption readers can see for example (Osborne, 2004) and (Dixit & Skeath, 2004).

1.4.1 General information about games

Most of the definitions in this chapter are from (Osborne, 2004). Let's assume that the payers playing the game are rational and thus want to maximize the payoff they can achieve. These are called *payoff functions* and they can be represented by a numerical value. For a player to act rationally between two choices a and b, the player prefers a if and only if the payoff function of a is greater than the payoff function of b. I'll use u to represent players preference over the possible outcomes. Thus we have

$$u(a) > u(b)$$

If the two outcomes have the same payoff, the player is indifferent between then and chooses both with the same probability.

1.5 Solving non-cooperative games

This chapter is going to cover the basics how to solve non-cooperative games. As previously stated most definitions come from Osborne(2004). While this chapter does give out basic tools on how to solve games quite a few different other tools will be left uncovered. This chapter covers how to solve non-cooperative games. The next chapter will cover how to solve simple cooperative games.

1.5.1 Normal form games

The definition of a strategic games in chapter 1.2 was the combination of three things. Players, actions and preferences over the possible outcomes. It assumed that both players make their decisions simultaneously. The game formed in chapter 1.3.2 where two criminals were under investigation. Both preferred going free to short sentence to medium sentence to long sentence Both players decide their actions simultaneously. This game is known as The Prisoners Dilemma and it was developed by Albert Tucker (Poundstone, 1992). There are two ways the game can be modeled. In normal form, where the players outcomes would be put in to a matrix or in strategic form where the outcomes would be put in as leaves of a decision tree. More detailed differences will be covered later. Now to model this situation as a normal form game. The previously formed games to be like this

Players: the two criminals
Strategies: both can choose between {Talk and Silent}
Preferences: going free > short sentence > medium sentence > long sentence

This somewhat resembles the game formed in 1.2. quite closely. In fact both games can be analyzed in normal form. Before analyzing this game payoff functions need to be added to the the preferences. Now to write the payoff functions so that player 1's choice is the first parameter and player 2's choice is the second parameter. Thus, for player 1

$$u_1(Talk, Silent) > u_1(Silent, Silent) > u_1(Talk, Talk) > u_1(Silent, Talk)$$

and for player 2

$$u_2(Silent, Talk) > u_2(Silent, Silent) > u_2(Talk, Talk) > u_1(Talk, Silent)$$

By choosing the following numerical values for player1

$$u_1(Talk, Silent) = 3, u_1(Silent, Silent) = 2, u_1(Talk, Talk) = 1, u_1(Silent, Talk) = 0$$

and for player 2

$$u_2(Silent, Talk) = 3, u_2(Silent, Silent) = 2, u_2(Talk, Talk) = 1\ u_1(Talk, Silent) = 0$$

the following game can be formed

		Criminal 2	
		Silent	Talk
Criminal 1	Silent	2 , 2	0 , 3
	Talk	3 , 0	1 , 1

According to chapter 1.4 all players act rationally and thus want to maximize their payoffs. By comparing the different payoffs it is clear that Talk always yields better payoff (3>2, 1>0). This is called Criminal 1' *best response* to Criminal 2's actions. For Criminal 2's Silent Criminal 1's best response is to choose Talk. For Criminal 2's action Talk Criminal 1's best response is to choose Talk. Therefore the rational thing to do is to always choose Talk. Playing Silent in any situation always gives an outcome that is suboptimal and can be improved by playing Talk. In this situation talk s*trictly dominates* Silent. In an equilibrium situation no player plays strictly dominated strategies. Because no rational player plays strictly dominated strategies they can be removed from the payoff matrix. From the matrix it is clear that Talk strictly dominates Silent. Therefore, Silent can be remove from both players action sets. This leaves on only one possible outcome (Talk, Talk). This means that if both players play rationally they will always play Talk and the game will always end in (Talk, Talk) with the payoffs of (1,1).

1.5.2 Best responce and Nash Equilibrium

John Forbes Nash formed probably the most important solving tool for games in his doctoral thesis. *Nash equilibrium of strategic games with ordinal preferences* expands on the ideas that we previously discussed and can be formulated as follows(Osbourne, 2004, 23).

> "...for every player *i* and every action a_i of player i, a^* is at least as good according to player *i* preferences as the action profile (a_i, a^*_{-i}) in which player *i* chooses a_i while every other player *j* chooses a^*_{-i}. Equivalently, for every player *i*, $u_i(a^*) \geqslant u_i(a_i, a^*_{-i})$, for every action a_i of player *i*, where u_i is a payoff function that represents player *i*'s preferences.

This means that if no player can improve their payoffs by chancing to another strategy unilaterally then the strategy played is a Nash equilibrium. From the

way that the definition is formed it is clear that there might be more than one Nash Equilibrium in a game. By examining the different strategies in the Prisoners Dilemma it is obvious that (Talk, Talk) is indeed a Nash equilibrium. What about the restaurant game. Does it have any Nash equilibrium?

- Players: the two friends
- Set of actions: each player has the choice between {restaunt1, restaurant2}
- Outcome: each one prefers to eat at her favorite restaurant

To give the outcomes an ordinal preference I have chosen $u_i(\textit{favorite restaurant})=2, u_i(\textit{other restaurant})=1$ and $u_i(\textit{eat alone})=0$. Both of the players have symmetrical preferences we have the following game.

		Player 2	
		Restaurant1	Restaurant2
Player1	Restaurant1	2 , 1	0 , 0
	Restaurant2	0 , 0	1 , 2

Clearly there are no strictly dominated strategies. By marking the best responses with a * for both players we have

		Player 2	
		Restaurant1	Restaurant2
Player1	Restaurant1	2 * , 1 *	0 , 0
	Restaurant2	0 , 0	1 * , 2 *

Now there are two cells in which both players have marked their best responses. Turns out that they both are in fact Nash equilibrium because they are the best responses to each others every strategy. Let $B_i(a_{-i})$ be the best response of player *i* to every other players action a_{-i}. From this the following can be formulated(Osborne, 2004, 36).

> The action profile a^* is a Nash equilibrium of a strategic game with ordinal preferences if and only if every player's action is a best response to the other players' action:
>
> a_i^* is in $B_i(a_{-i}^*)$ for every player *i*

From this it is obvious that by inspecting every cell and marking the best responses for every action for every player *i* it is possible find Nash equilibrium in games where there are no dominant strategies.

1.5.3 Mixed strategies

Let's look at a game where there are exits no Nash equilibrium. Classically called "Matching pennies". Two players are playing a game where they decide on whether to heads or tails of a penny. The decisions are made simultaneously. If the players show the same side player 2 pays player 1 one euro. If the players show different sides then player 1 pays player 2 1 euro. Both prefer to receive money to losing money. To make this in to normal form game

		Player 2	
		Head	Tail
Player1	Head	1, -1	-1,1
	Tail	-1,1	1,-1

Clearly there are no dominant strategies and the best responses are placed as follows.

		Player 2	
		Head	Tail
Player1	Head	*1, -1	-1,*1
	Tail	-1,*1	*1,-1

It appears that there are no Nash equilibrium in this game. However, there is a way to solve this game if all of the potential outcomes are thought to be expected payoffs and played with a certain probability. From this it is possible to calculate Nash equilibrium in the game of Matching Pennies. The preferences in situation are called *vNM preferences* after von Neumann and Morgenstern(1944) who studied preferences over lotteries. Osborne (2004, 108) gives out the following definition for *mixed strategy Nash equilibrium of strategic game with vNM preferences*.

> The mixed strategy profile a^* in a strategic game with ordinal preferences, is a (mixed strategy) Nash equilibrium if, for each player *i* and every mixed *a* strategy α_i of player *i*, the expected payoff to player *i* of α_i is at least as large as the expected payoff to player *i* of (α_i, α^*_i) according to a payoff function whose expected value represents players *i*'s preferences over lotteries. Equivalently, for each player *i*,

$U_i(\alpha^*) \geqslant U_i(\alpha_i, \alpha^*_{-i})$ for every mixed strategy α_i of player *i*, where $U_i(\alpha)$ is player *i*'s expected payoff to the mixed strategy profile α.

It also turns out that the Nash equilibrium discussed before is a special case of the mixed strategy version and are usually called pure strategies. Now to analyze Matching Pennies. Everything stays the same except the different strategies are assigned probabilities. Player 1 will play Head with the probability p and Tail with the probability $(1-p)$. Player 2 will play Head with the probability q and Tail with the probability $(1-q)$.

		Player 2: Head (q)	Player 2: Tail $(1-q)$
Player1	Heads (p)	1, -1	-1, 1
	Tails $(1-p)$	-1, 1	1, -1

the mixed strategy Nash equilibrium in this game can be found when the both players expected payoffs are maximal. For player 1 this can be done when

$$pq+(1-q)(-p)+(1-p)(-q)+(1-p)(1-q)=4\text{pq}-2\text{p}-2\text{q}+1=p(4\text{q}-2)-2\text{q}+1$$

is maximal. From $p(4\text{q}-2)-2\text{q}+1$ it is clear that to maximize p the following best response equations exist

$$B_1(q)=\begin{cases}4\text{q}-2>0\\4\text{q}-2<0\\4\text{q}-2=0\end{cases} \text{and solved } B_1(q)=\begin{cases}(q>1/2)\rightarrow p=1\\(q<1/2)\rightarrow p=0\\(q=1/2\rightarrow p:0\leqslant p\leqslant 1)\end{cases}$$

For player 2's payoffs

$$-pq+p(1-q)+(1-p)q-(1-p)(1-q)=-4\text{pq}+2\text{p}+2\text{q}-1=q(-4\text{p}+2)+2\text{p}-1\,.$$

To maximize for q from $q(-4\text{p}+2)+2\text{p}-1$ it is clear that player 2's best responses are

$$B_2(p)=\begin{cases}-4\text{p}+2>0\\-4\text{p}+2<0\\-4\text{p}+2=0\end{cases} \text{and solved } B_2(p)=\begin{cases}(p<1/2)\rightarrow q=1\\(p>1/2)\rightarrow q=0\\(p=1/2\rightarrow q:0\leqslant q\leqslant 1)\end{cases}$$

To plot the results to a coordinate the following diagram, where player 1 is back and player 2 is gray

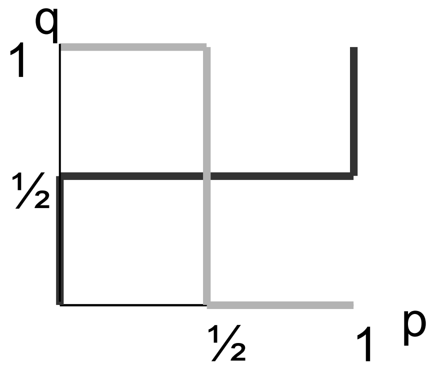


Figure 1 Players best responses

From the figure it is clear that the players best plots intersect only in one point $(p,q)=(\frac{1}{2},\frac{1}{2})$ and that is the Nash equilibrium of this game. This means that both players should play Head with probability $\frac{1}{2}$ and Tail with probability $\frac{1}{2}$. This approach can be applied to the previous Restaurant game and a previously unknown Nash equilibrium point $(\frac{2}{3},\frac{1}{3})$ will be the result.

1.5.4 Repeated games & Folk theorem

As previously discussed the Prisoners Dilemma has a dominant strategy of Talk. Thus if both players act rationally both will always play Talk. In fact this is a Nash equilibrium. How about the if both players play Silent? If one looks closely at the game it can also be seen that (Silent, Silent) is also a Nash equilibrium. Therefore the game has two Nash equilibria. However is it possible for the players to actually get the payoffs from (Silent, Silent) as Talk always yields better results? This is where repeated games come in. If the players interact more than once the game might have a different outcome.

Let's say that the two players choose to play Silent in the first round then it is easy to see that the final payoffs are going to be greater than choosing Talk $(3>2)$. It is easy to see that the payoffs can be thought to be sums of all the individual rounds. It is assumed that players associate a *discount sum* to the payoffs. From these premises the following can be said(Osborne, 2004, 421)

> ...each player I has a payoff function u_i for the strategic game and a discount factor δ_i between 0 and 1 such that she evaluates the sequence $(a^{1,}a^{2,}...,a^{T})$ of

outcomes of the strategic game by the sum $u_i(a^1)+\delta_i u_i(a^2)+\delta_i^2 u_i(a^3)+\cdots+\delta_i^{T-1} u_i(a^T)=\sum_{t=1}^{T}\delta_i^{t-1}u_i(a^t)$ where

a^t is the action profile in period t and δ_i^t is the discount factor δ_i raised to the power t.

Now it is possible to show that by repeating games different strategies can yield a better payoff than the one given by a Nash equilibrium in a game that is played once.

It is also clear that if the players cooperate even once their payoffs will be greater than playing Silent in every turn. Osborne(2004, 435) describes the Nash folk theorem for infinitely repeated Prisoner's Dilemma that can be used when analyzing repeated games. This is know as the *Folk Theorem* and it describes how Nash equilibrium can be found in repeated games. Due to the technicality of the Folk theorem it is suggested that interested readers look at the previously mentioned book. The folk theorem can be used as a base for a strategy when playing repeated games.

1.6 Solving cooperative games

Cooperative games are games in which the players will cooperate in order to improve their outcomes. The only way players will act in this way is to make sure that the payoff from cooperating in greater than acting alone. It is assumed that binding agreements can be made and that players can be forced by an outside party to act as they have agreed. As in previous chapter, this chapter is also based on Osborne(2004). After seeing some simple cooperative games tools to solve them will be presented. Not all types of cooperative games will be covered. Therefore, the reader should point their attention to the previously mentioned book by Osborne or to Dixit & Skeath(2004).

1.6.1 coalitional games and transferable utility

In cooperative games players work together to improve their payoffs. There are numerous different ways players can cooperate e.g. individually or in a group. This chapter show that if players form coalitions they can achieve greater payoffs when compared to acting alone. These types of games are called *coalitional games* and they are defined as (Osborne, 2004)

-A set of players
-for each coalition, a set of actions
-for each player, preferences over the set of all actions of all coalitions of which she is a member

It should be noted that coalitions can range from a single player to a coalition of all the players which is called a *grand coalition.*

Possibly the most simple cooperative game is a game where two players need to cooperate in order to get 1 €. Neither can get the 1€ by working alone. Both players prefer to get more money to getting less money. Therefore we have the following game (Osborne 2004, 240)

> -*Players* The two workers (players 1 and 2)
> -*Actions* Both players have a single action which yields the player no output. The set of actions of the coalition {1,2} of both players is the set of all pairs $(x_{1,} x_2)$ of nonnegative numbers such that $x_1+x_2=1$ (the set of divisions of the one unit of output between the two players.
> -*Preferences* Each player's preferences are represented by the output the player obtains.

It is clear that both players need to participate in the coalition, which happens to be a *grand coalition,* in order to receive anything. A grand coalition is usually denoted by N and a coalition of any other type is usually denoted by S. The payoff can be divided amongst the players of the coalition. These types of games where the total payoff can be divided amongst the players in the coalition are called *coalitional games with transferable utility* and they are formally defined as follows(Osborne, 2004, 241)

> A coalitional game has transferable payoff if there is a collection of payoff functions, one representing each player's preferences, such that for each coalition S, every action of S generates a distribution of payoffs among the members of S that have the same sum.

This is commonly denoted as $v(S)$ and it is usually called the *worth* of a coalition. From this the previously presented game where two players need to cooperate to receive 1€ the payoffs can be represented as follows $N=1,2, v(1)=v(2)=0, v(1,2)=1$.

1.6.2 The core & shapley value

How does a coalition decide what it should do? Is it possible to predict how a coalition will behave? As previous chapter has shown there are certain properties of a coalition that can give a clue how any coalition should act. An action coalition is defined to be *stable* if no coalition can break away from the bigger coalition because its members prefer another action. Osborne (2004, 243) defines the set of all stable actions by a grand coalition as the *core*. In a game there always exists a core but the core can be defined to be *empty*. This happens when in any situation some coalition S can receive a better payoff by acting differently than the rest of the coalition. In this situation it is said that S can *improve upon* a_N, where a_N is the set of actions by the grand coalition N. Thus it can be said that (Osborne, 2004, 244)

> a_N is in the core of a coalitional game with transferable payoff if and only if for every coalition S the total payoff $x_S(a_N)$ it yields the members if S is at least $v(S)$: $x_S(a_N) \geqslant v(S)$ for every coalition S.

From this it is clear that for the game in previous chapter the exists a core and that the core consists of all possible divisions. That is because if one of the players deviates form the actions of the grand coalition N that player receives the payoff of 0.

In order to divide the payoffs amongst the players within any coalition there needs to be a way to do it fairly. This is where the *Shapley value* comes handy. Named after Lloyd Shapley it can be used to divide the payoff of a coalition so that every player receives a a payoff that is equal to the amount that the player contributed to the coalition. Dixit & Skeath (2004, 619) define the value as follwos

$$u_i = \sum_C \frac{(n-k)!(n-1)!}{n!}[v(S)-v(S-i)]$$

where n is the number of coalitions and k is the number of players in the coalition. To look at a real life application of the Shapley value (Dixit & Skeath, 2004, 260-262).

Let's say that there exist a 100-member legislature consisting of four parties. The parties are called Red, Blue, Green, Brown. The Reds have 43 seats, The Blues have 33 seats, The Greens have 16 seats and the browns have 8 seats. Each member of the party votes with the other members of her party. As there are four parties there exists four one party coalitions which all are trivial as none of

them can gain majority by themselves. There are three two-party coalitions that can for a majority. These are {Red, Blue}, {Red, Green} and {Red, Brown}.Of the possible three-party coalitions, {Red, Blue, Green}, {Red, Blue, Brown}, {Red, Green, Brown}, red is pivotal. That is to say that with out the Red the coalition can not achieve majority. In the three-party coalition {Blue, Green, Brown] all three parties are pivotal. For every two-party coalition get's value

$$\frac{(4-2)!(2-1)!}{4!}=\frac{1}{12}$$

and for every three-party

$$\frac{(4-3)!(3-1)}{1!}=\frac{1}{12}$$

in the Shapley value formula. When this information is known it is possible to calculate the Shapley value for each party. Thus we have

$$u_{Red}=\frac{1}{12}\times 3+\frac{1}{12}\times 3=\frac{1}{2}$$

for the Red party is pivotal in three two-party coalitions and in three three-party coalitions. Shapley value can be calculated also for the other parties.

$$u_{Blue}=U_{Green}=u_{Brown}=\frac{1}{12}\times 1+\frac{1}{12}\times 1=\frac{1}{6}$$

While the Blue party might over twice the members of the Green party the three parties have the same amount of power. The reasons for this is that they all play key roles in forming majority coalitions. They can either join Red party to form a majority or join in with the other two parties to form a three-party majority. If for example the two biggest parties for some reason cannot form a coalition the power of the smaller parties increases.

Now that the basic tools how to solve different types of cooperative and non-co-operative games have been covered, the next chapter will give the reader an overlook into how wireless devices work and how the tools that game theory provides can be used to analyze the interaction within them.

A really brief introduction of Wireless technology

Future technology and what limitations it will have. Talk about cognitive radios and stuff.
Verkkoihin taustaa, miksi peliteoriaa voidaan käyttää, tulevaisuutta,

Game Theory in wireless networks

Why wireless networks can be analyzed with game theory? It seems that there are no connections between the abstract ideas of von Neumann and Nash and your favorite hand held device.

1.7 Cooperative games in wireless networks

Cooperative games in wireless networks all have one assumption in common. They all need to have some way of enforcing agreements that are made. This can be done by making standards within the industry or by having an outside institution enforce the agreements. The outside institution can be the FCC, for example. Unfortunately, if agreements can not be forced then the game becomes a non-cooperative game. Another threat to these types of networks where nodes cooperate is that if a non-cooperative node participates in the game, it can free ride on the results of others without making any contributions to the network. Therefore, most games that are going to be presented in this thesis are going to be non-cooperative games.

1.7.1 Different types of cooperative games

As stated in the previously cooperative games need a way to enforce agreements and when that requirement is fulfilled there are numerous scenarios that can be modeled. These include, but are not limited to, channel access time, spectrum assignment, channel allocation, packet forwarding and spectrum sensing (Maharjan, Zhang & Gjessing 2010; Charilas, Panagopoulos 2010).

1.8 Non-cooperative Games in wireless networks

Stuff about non-cooperative games in wireless networks. Forwarder's dilemma(cooperation in scarcity), folk theorem and strategies(Always cooperate, always defect, TFT, Stochastic-TFT, Expected utility).

Summary

summary about what you just red, where this findings can be applied and future research.

REEFERENCES

Guess what?

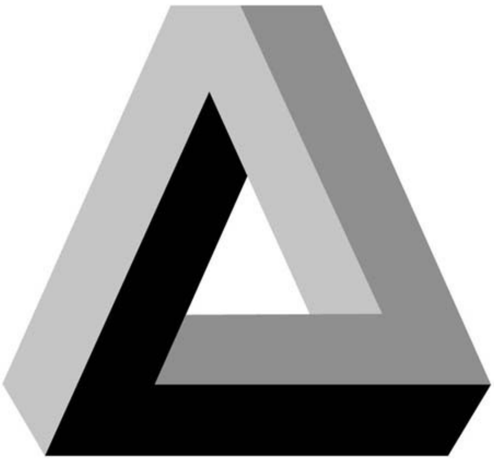
LIITE 1 ENSIMMÄINEN LIITE

Tämä on tutkielman mahdolliselle liitteelle varattu sivu. Mikäli liitteitä on useampia, ne sijoitetaan omille sivuilleen. Eli jos tässä raportointipohjassa olisi liite kaksi, se alkaisi omalta sivultaan.

TAULUKKO 1 Esimerkki taulukon selitetekstistä

Dokumenttityyppi	**Muistio**	**Lasku**
Organisaation logo	Ei esiinny	Esiintyy
Yhteystiedot	Esiintyy	Esiintyy
Laatijan nimi	Esiintyy	Ei esiinny

Seuraavassa esimerkki kuviosta mahdoton kolmio (kuvio 1):



KUVIO 2 Mahdoton kolmio