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NON-COOPERATIVE GAME THEORY IN WIRELESS NETWORKS

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Asiasanat: asiasana1, asiasana2, asiasana3, …

ABSTRACT

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1. Introduction

Some general information about why you should read this thesis, where you could find game theory.

introon esimerkki ja miten verkot ja peliteoria yhdistetään, miksi hyödyllistä. Esitellään myös tutkimusongelmat

A brief overview of game theory

Merriam-Webster defines game theory, as "the analysis of a situation involving conflicting interests (as in business or military strategy) in terms of gains and losses among opposing players". What this means is that game theory is used to describe decision making situations and the interaction that happens. For this, the mathematical tools that game theory provides are a valuable asset. Due to the fact that the tools that game theory provides are quite abstract, they can be used in various situations ranging from the strictly mathematical to social sciences and information technology. In recent years there has even been some advances in to using game theory to analyze human decision making in a field called Neuroeconomics {{33 Glimcher, P.W. 2004;32 Camerer, C. 2005;}}. The next chapter will give a more detailed description in to the history of Game Theory.

1.1 History

Game theory has a very long history and the first results of game theoretic modeling can be seen in the Talmud{{36 Aumann,Robert J. 1985;}} where the results of bankruptcy are considered. Augustin Cournot (1801 – 1877) formed a model of oligopoly{{37 Cournot, A.A. 1838;}} which models the interaction between a small number of sellers. Currently, the field of game theory is considered to have gotten formed in 1944 by the publication of "Theory of games and economic behavior" by John von Neumann and Oskar Morgenster {{39 Von Neumann, J. 1944;}}. The book has essential tools to solve games such as backwards induction. As for games the book features zero-sum games, non-zero sum games as well as games with perfect and imperfect information. Most of these are covered later in this thesis.

Next big step froward was when developed by John Forbes Nash. Nash's doctoral thesis included the definition for the Nash Equilibrium. The thesis served as a foundation for four articles that include non-cooperative games {{40 Nash, J. 1951;}} and bargaining {{41 Nash Jr, J.F. 1950;}}. For these developments Nash was awarded the Nobel Memorial Price in Economic Sciences in 1994. The price was awarded together with Reinhard Selten and John Harsanyi. All together the Nobel Price has been awarded to eight game theorists. In in recent years game theory has been used as a base of mechanism design {{45 Fudenberg, D. ;}}. This field is sometimes called reverse game theory.

1.2 What is a strategic game?

In a game strategic there are different actors who have to make decisions on how to act upon certain rules and have certain preferences over out comes. This means for example that a situation when two friends are deciding on where to eat and they both have a favorite restaurant can be modeled as a strategic game. The definition of such a strategic game is as follows {{18 Osborne, M.J. 2004;}}

- a set of players
- each player has a set of actions
- each player has some preferences over the possible outcomes

In this the players are the two friends. Both of them can choose where they will eat but would prefer to go each own favorite restaurant. The game derived from these premises is usually called the “battle of the sexes” or “Bach or Stravisky”.

- Players: the two friends
- Set of actions: each player has the choice between {restaunt1, restaurant2}
- Outcomes: each one prefers to eat at her favorite restaurant

We will analyze this game later.

1.3 Different types of games

There are many ways one could categorize different types of games. Avinash Dixit and Susan Skeath gave a very good list of different types of games in their

book “Games of Strategy”. In the book they divide games in to six different categories ({{19 Dixit, A.K. 2004;}}, 20-27)

- Do the players take turns? If they do the game is called a sequential game and the players make their decision simultaneously the game is called a simultaneous games.
- Zero sum games, that is are the players interests opposite or do they share some interests.
- Repeated games. Is the game going to be played more than once. If played only once then the game is called a one-shot game. If played more than once then the game is called a repeated game.
- Games of perfect information. Do the players know everything about their opponents or is there uncertainty in the game? If there is uncertainty in the game then it is called a game of imperfect information. If one player know more about the game than her opponent(s) then the game is called a game with incomplete, or asymmetric, information.
- Can the rules be manipulated? If the rules can be manipulated then the pregame becomes the real game as this will surely effect the outcome of the game.
- Can cooperation agreements be enforced? If agreements indeed can be enforced then the game is called a cooperative game. If not then the game is called a non-cooperative game.

The games that are going to be looked at can be divided in to two main categories: Cooperative games and non-cooperative games. The reason for this is quite simple because the reasoning and logic behind cooperative game is different than the reasoning behind non-cooperative. In cooperative games the players are trying to maximize their outcomes by working as a group in an environment where agreements can be enforced. This is quite the opposite in non-cooperative games where agreements can not be enforced and therefore it is in every players interest to maximize her payoff. One other reason for dividing them in to these two categories is that both cooperative and non-cooperative games can have games with the other properties listed above within the games themselves. This is to say that there can be a repeated non-cooperative game with imperfect information. That is also going to be one of the examples that is going to be discussed in the section of non-cooperative games in wireless networks.

1.3.1 Cooperative Games

Games of cooperation are games where agreements enforceable {{19 Dixit, A.K. 2004;}}. This means that the players decisions are made in a group and that all

members of the group will act according to the decision or a game where all players act according the agreements that can be forced collectively or directly. By doing so the players cooperate to maximize their payoffs. Working together they can for example form coalitions that aim to increase the payoff of the coalition and thus increase the payoff to each member.

1.3.2 Non-cooperative Games

Non-cooperative games are games where agreements can not be forced and thus the main focus of each player is to maximize their own payoffs. While this does make it seem like every player always acts selfish it is not always the case. In a later chapter there will be an example of a game where cooperation still guarantees the best outcome. Probably the most famous non-cooperative game is called the prisoners dilemma. The ideas behind prisoners dilemma is quite simple. Two friends have committed a crime and been detained by the police. The police have enough information on the two to convict them because of a smaller crime that two did earlier. They also know that the two did a larger crime but they lack the information to prosecute. The police make an offer where the person who rats out the accomplice gets to walk free but the other person gets a longer sentence. If both of them talk then both of them will serve a long sentence. Both prefer walking free to small sentence to a long sentence. This is also how her partner in crime thinks. Therefore, this situation can be formed as the following game

Payers: the two criminals.
Strategies: {talk, don't talk}
Preferences: walk free > short sentence > medium sentence > long sentence

The next chapter provides everything needed in order to analyze and solve this situation.

1.4 How are games solved?

In this chapter covers how different types of games can be analyzed and solved. Due to the fact that this chapter is going to be notation heavy there will be quite a few examples. For more advanced readers should look at {{18 Osborne, M.J. 2004;}} as it contains more formal definitions of the following games. For a reader wanting more explanations {{19 Dixit, A.K. 2004;}} might be good be-

cause it contains more thorough explanations and doesn't have as much formal equations as former.

Game theory assumes that the players are rational and acts rationally to maximize their payoffs {{18 Osborne, M.J. 2004;}}. What this means is that the player can assign some kind of values to different outcomes of the games and act rationally to maximize it. Players are assumed to have some common knowledge. For more thorough commentary on what rationality implies, ({{18 Osborne, M.J. 2004;}}, 6-7; {{19 Dixit, A.K. 2004;}}, 29 - 32) is a good place to start. For criticism on the rationality assumption readers can see for example {{18 Osborne, M.J. 2004;}} and {{19 Dixit, A.K. 2004;}}.

1.4.1 General information about games

Most of the definitions in this chapter are from {{18 Osborne, M.J. 2004;}}. Let's assume that the payers playing the game are rational and thus want to maximize the payoff they can achieve. These are called *payoff functions,* or *utility functions,* and they can be represented by a numerical value. For a player to act rationally between two choices a and b, the player prefers a if and only if the payoff function of a is greater than the payoff function of b. I'll use u to represent players preference over the possible outcomes. Thus we have

$$u(a) > u(b)$$

If the two outcomes have the same payoff, the player is indifferent between then and chooses both with the same probability.

1.5 Solving non-cooperative games

This chapter is going to cover the basics how to solve non-cooperative games. As previously stated most definitions come from {{18 Osborne, M.J. 2004;}}. While this chapter does give out basic tools on how to solve games quite a few different other tools will be left uncovered. This chapter covers how to solve non-cooperative games. The next chapter will cover how to solve simple cooperative games.

1.5.1 Normal form games

The definition of a strategic games in chapter 1.2 was the combination of three things. Players, actions and preferences over the possible outcomes. It assumed that both players make their decisions simultaneously. The game formed in chapter 1.3.2 where two criminals were under investigation. Both preferred going free to short sentence to medium sentence to long sentence Both players decide their actions simultaneously. This game is known as The Prisoners Dilemma and it was developed by Albert Tucker {{43 Poundstone, W. 1992;}}. There are two ways the game can be modeled. In normal form, where the players outcomes would be put in to a matrix or in strategic form where the outcomes would be put in as leaves of a decision tree. More detailed differences will be covered later. Now to model this situation as a normal form game. The previously formed games to be like this

Players: the two criminals
Strategies: both can choose between {Talk and Silent}
Preferences: going free > short sentence > medium sentence > long sentence

This somewhat resembles the game formed in 1.2. quite closely. In fact both games can be analyzed in normal form. Before analyzing this game payoff functions need to be added to the the preferences. Now to write the payoff functions so that player 1's choice is the first parameter and player 2's choice is the second parameter. Thus, for player 1

$$u_1(Talk, Silent) > u_1(Silent, Silent) > u_1(Talk, Talk) > u_1(Silent, Talk)$$

and for player 2

$$u_2(Silent, Talk) > u_2(Silent, Silent) > u_2(Talk, Talk) > u_1(Talk, Silent)$$

By choosing the following numerical values for player1

$$u_1(Talk, Silent) = 3, u_1(Silent, Silent) = 2, u_1(Talk, Talk) = 1, u_1(Silent, Talk) = 0$$

and for player 2

$$u_2(Silent, Talk) = 3, u_2(Silent, Silent) = 2, u_2(Talk, Talk) = 1\ u_1(Talk, Silent) = 0$$

the following game can be formed

		Criminal 2	
		Silent	Talk
Criminal 1	Silent	2,2	0,3
	Talk	3,0	1,1

According to chapter 1.4 all players act rationally and thus want to maximize their payoffs. By comparing the different payoffs it is clear that Talk always yields better payoff (3>2, 1>0). This is called Criminal 1' *best response* to Criminal 2's actions. For Criminal 2's Silent Criminal 1's best response is to choose Talk. For Criminal 2's action Talk Criminal 1's best response is to choose Talk. Therefore the rational thing to do is to always choose Talk. Playing Silent in any situation always gives an outcome that is suboptimal and can be improved by playing Talk. In this situation talk s*trictly dominates* Silent. In an equilibrium situation no player plays strictly dominated strategies. Because no rational player plays strictly dominated strategies they can be removed from the payoff matrix. From the matrix it is clear that Talk strictly dominates Silent. Therefore, Silent can be remove from both players action sets. This leaves on only one possible outcome (Talk, Talk). This means that if both players play rationally they will always play Talk and the game will always end in (Talk, Talk) with the payoffs of (1,1).

1.5.2 Best responce and Nash Equilibrium

John Forbes Nash formed probably the most important solving tool for games in his doctoral thesis. *Nash equilibrium of strategic games with ordinal preferences* expands on the ideas that we previously discussed and can be formulated as follows ({{18 Osborne, M.J. 2004;}}, 23).

> "...for every player *i* and every action a_i of player i, a^* is at least as good according to player *i* preferences as the action profile (a_i, a^*_{-i}) in which player *i* chooses a_i while every other player *j* chooses a^*_{-i}. Equivalently, for every player *i*, $u_i(a^*) \geqslant u_i(a_i, a^*_{-i})$, for every action a_i of player *i*, where u_i is a payoff function that represents player *i*'s preferences.

This means that if no player can improve their payoffs by chancing to another strategy unilaterally then the strategy played is a Nash equilibrium. From the

way that the definition is formed it is clear that there might be more than one Nash Equilibrium in a game. By examining the different strategies in the Prisoners Dilemma it is obvious that (Talk, Talk) is indeed a Nash equilibrium. What about the restaurant game. Does it have any Nash equilibrium?

- Players: the two friends
- Set of actions: each player has the choice between {restaunt1, restaurant2}
- Outcome: each one prefers to eat at her favorite restaurant

To give the outcomes an ordinal preference I have chosen $u_i(\textit{favoriterestaurant})=2, u_i(\textit{other restaurant})=1$ and $u_i(\textit{eat alone})=0$. Both of the players have symmetrical preferences we have the following game.

		Player 2	
		Restaurant1	Restaurant2
Player1	Restaurant1	2,1	0,0
	Restaurant2	0,0	1,2

Clearly there are no strictly dominated strategies. By marking the best responses with a * for both players we have

		Player 2	
		Restaurant1	Restaurant2
Player1	Restaurant1	2*,1*	0,0
	Restaurant2	0,0	1*,2*

Now there are two cells in which both players have marked their best responses. Turns out that they both are in fact Nash equilibrium because they are the best responses to each others every strategy. Let $B_i(a_{-i})$ be the best response of player *i* to every other players action a_{-i}. From this the following can be formulated ({{18 Osborne, M.J. 2004;}}, 36).

> The action profile a^* is a Nash equilibrium of a strategic game with ordinal preferences if and only if every player's action is a best response to the other players' action:
>
> a_i^* is in $B_i(a_{-i}^*)$ for every player *i*

From this it is obvious that by inspecting every cell and marking the best responses for every action for every player *i* it is possible find Nash equilibrium in games where there are no dominant strategies.

1.5.3 Mixed strategies

Let's look at a game where there are exits no Nash equilibrium. Classically called "Matching pennies". Two players are playing a game where they decide on whether to heads or tails of a penny. The decisions are made simultaneously. If the players show the same side player 2 pays player 1 one euro. If the players show different sides then player 1 pays player 2 1 euro. Both prefer to receive money to losing money. To make this in to normal form game

		Player 2: Head	Player 2: Tail
Player1	Head	1, -1	-1,1
	Tail	-1,1	1,-1

Clearly there are no dominant strategies and the best responses are placed as follows.

		Player 2: Head	Player 2: Tail
Player1	Head	*1, -1	-1,*1
	Tail	-1,*1	*1,-1

It appears that there are no Nash equilibrium in this game. However, there is a way to solve this game if all of the potential outcomes are thought to be expected payoffs and played with a certain probability. From this it is possible to calculate Nash equilibrium in the game of Matching Pennies. The preferences in situation are called *vNM preferences* after von Neumann and Morgenstern(1944) who studied preferences over lotteries. {{18 Osborne, M.J. 2004;}}, 108 gives out the following definition for *mixed strategy Nash equilibrium of strategic game with vNM preferences*.

> The mixed strategy profile a^* in a strategic game with ordinal preferences, is a (mixed strategy) Nash equilibrium if, for each player *i* and every mixed *a* strategy α_i of player *i*, the expected payoff to player *i* of α_i is at least as large as the expected payoff to player *i* of (α_i, α^*_i) according to a payoff function whose expected value represents players *i*'s preferences over lotteries. Equivalently, for each player *i*,

$U_i(\alpha^*) \geqslant U_i(\alpha_i, \alpha_{-i}^*)$ for every mixed strategy α_i of player *i*, where $U_i(\alpha)$ is player *i*'s expected payoff to the mixed strategy profile α .

It also turns out that the Nash equilibrium discussed before is a special case of the mixed strategy version and are usually called pure strategies. Now to analyze Matching Pennies. Everything stays the same except the different strategies are assigned probabilities. Player 1 will play Head with the probability p and Tail with the probability $(1-p)$. Player 2 will play Head with the probability q and Tail with the probability $(1-q)$.

		Player 2 Head (q)	Player 2 Tail $(1-q)$
Player1	Heads (p)	1, -1	-1,1
	Tails $(1-p)$	-1,1	1,-1

the mixed strategy Nash equilibrium in this game can be found when the both players expected payoffs are maximal. For player 1 this can be done when

$$pq+(1-q)(-p)+(1-p)(-q)+(1-p)(1-q)=4\mathrm{pq}-2\mathrm{p}-2\mathrm{q}+1=p(4\mathrm{q}-2)-2\mathrm{q}+1$$

is maximal. From $p(4\mathrm{q}-2)-2\mathrm{q}+1$ it is clear that to maximize p the following best response equations exist

$$B_1(q)=\begin{cases}4\mathrm{q}-2>0\\4\mathrm{q}-2<0\\4\mathrm{q}-2=0\end{cases} \text{ and solved } B_1(q)=\begin{cases}(q>1/2)\rightarrow p=1\\(q<1/2)\rightarrow p=0\\(q=1/2\rightarrow p:0\leqslant p\leqslant 1)\end{cases}$$

For player 2's payoffs

$$-pq+p(1-q)+(1-p)q-(1-p)(1-q)=-4\mathrm{pq}+2\mathrm{p}+2\mathrm{q}-1=q(-4\mathrm{p}+2)+2\mathrm{p}-1 \text{ .}$$

To maximize for q from $q(-4\mathrm{p}+2)+2\mathrm{p}-1$ it is clear that player 2's best responses are

$$B_2(p)=\begin{cases}-4\mathrm{p}+2>0\\-4\mathrm{p}+2<0\\-4\mathrm{p}+2=0\end{cases} \text{ and solved } B_2(p)=\begin{cases}(p<1/2)\rightarrow q=1\\(p>1/2)\rightarrow q=0\\(p=1/2\rightarrow q:0\leqslant q\leqslant 1)\end{cases}$$

To plot the results to a coordinate the following diagram, where player 1 is back and player 2 is gray

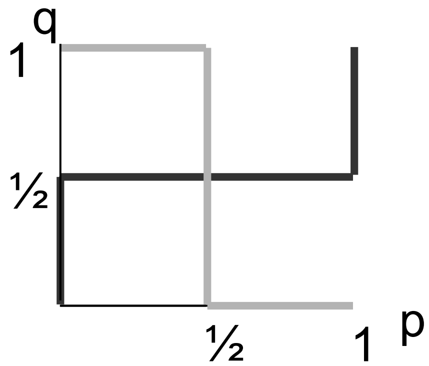

Figure 1 Players best responses

From the figure it is clear that the players best plots intersect only in one point $(p,q)=(\frac{1}{2},\frac{1}{2})$ and that is the Nash equilibrium of this game. This means that both players should play Head with probability $\frac{1}{2}$ and Tail with probability $\frac{1}{2}$. This approach can be applied to the previous Restaurant game and a previously unknown Nash equilibrium point $(\frac{2}{3},\frac{1}{3})$ will be the result.

1.5.4 Repeated games & Folk theorem

As previously discussed the Prisoners Dilemma has a dominant strategy of Talk. Thus if both players act rationally both will always play Talk. In fact this is a Nash equilibrium. How about the if both players play Silent? If one looks closely at the game it can also be seen that (Silent, Silent) is also a Nash equilibrium. Therefore the game has two Nash equilibria. However is it possible for the players to actually get the payoffs from (Silent, Silent) as Talk always yields better results? This is where repeated games come in. If the players interact more than once the game might have a different outcome.

Let's say that the two players choose to play Silent in the first round then it is easy to see that the final payoffs are going to be greater than choosing Talk $(3>2)$. It is easy to see that the payoffs can be thought to be sums of all the individual rounds. It is assumed that players associate a *discount sum* to the payoffs. From these premises the following can be said ({{18 Osborne, M.J. 2004;}}, 421)

> ...each player I has a payoff function u_i for the strategic game and a discount factor δ_i between 0 and 1 such that she evaluates the sequence $(a^{1,}a^{2}\ldots,a^{T})$ of outcomes

of the strategic game by the sum $u_i(a^1)+\delta_i u_i(a^2)+\delta_i^2 u_i(a^3)+\cdots+\delta_i^{T-1} u_i(a^T)=\sum_{t=1}^{T} \delta_i^{t-1} u_i(a^t)$ where

a^t is the action profile in period t and δ_i^t is the discount factor δ_i raised to the power t.

Now it is possible to show that by repeating games different strategies can yield a better payoff than the one given by a Nash equilibrium in a game that is played once.

It is also clear that if the players cooperate even once their payoffs will be greater than playing Silent in every turn. ({{18 Osborne, M.J. 2004;}}, 435) describes the Nash folk theorem for infinitely repeated Prisoner's Dilemma that can be used when analyzing repeated games. This is know as the *Folk Theorem* and it describes how Nash equilibrium can be found in repeated games. Due to the technicality of the Folk theorem it is suggested that interested readers look at the previously mentioned book. The folk theorem can be used as a base for a strategy when playing repeated games.

A really brief introduction of Wireless technology

Future technology and what limitations it will have. Talk about cognitive radios and stuff.

Verkkoihin taustaa, miksi peliteoriaa voidaan käyttää, tulevaisuutta,

SINR!! cognitive radio

Game theory in wireless networks

There are many ways networks can be modeled but the requirements of game theory propose that most of the situations can be divided in to two main categories. Cooperative and non-cooperative games. As previously presented the main difference between these two is the fact if agreements can be forced or not. In wireless networks the outside enforcer can be the FCC, for example. If this is possible then the game can be thought to be a cooperative game. Otherwise the game can be considered to be a non-cooperative game.

The previous chapter explained the fundamental requirements are there so that the tools that game theory provides can be applied. From this it is clear that game theory can provide an inside into how interaction between wireless devices can be modeled and analyzed. This chapter will discuss the applications of non-cooperative games theory in the field of wireless networks. Cooperative games are not discussed in this thesis but the reader can find a good tutorial in {{2 Saad,W. 2009;}}.

As previously discussed wireless devices need to operate within a limited spectrum that needs to be utilized as well as possible. If few nodes use all of the bandwidth available the network simply will not work. All nodes transmitting on full power and when ever they want to is not only bad for the power consumption of the device it is also bad for the other nodes as the amount of collisions will rise and thus the need for retransmissions. Devices, or nodes, need to consider what other nodes do before they transmit. That is why devices need to consider what other devices do when for example deciding transmission power or when to transmit signal.

1.6 Non-cooperative game theory in wireless networks

Non-cooperative game theory can be used in various situations when analyzing the interactions within wireless networks. {{17 Charilas,Dimitris E. ;}} approach this question from the point of view of OSI layers. It the article Charilas & Panagopoulos give examples how game theory can be used to solve problems at different OSI layers.

OSI Layer	Application field	Specific application
Physical	Power control	Power control for CDMA
		Power control in OFDMA Networks
	Spectrum allocation	Spectrum sharing- Spectrum transactions
	MIMO Systems	Power management in MIMO
	Cooperative communications	Decode-and-forward cooperation
Data link	Medium access control	Access to slotted Aloha
		Random access to the interference channel
Network	Routing	Routing and forwarding
Transport	Call admission control	Request distribution among providers
		Call acceptance based on provider and customer context
	Load control	Termination of sessions based on provider and customer context
	Cell selection	Inter-cell and intra-cell games

Table 1Layered presentation of game theory applications {{17 Charilas,Dimitris E. ;}}

From the figure above it is clear that the applications of game theory are quite varied. {{4 Mehta,Saurabh 2009;}} also provide a list of typical problems where game theory can be used. {{4 Mehta,Saurabh 2009;}} and {{17 Charilas,Dimitris E. ;}} both have listed power control and spectrum sharing as well as routing in the problems that can be analyzed. In the following chapters these and a few other examples are going to be analyzed more closely.

1.6.1 Power control

Power control is a key concept in the wireless networks as it sets a few limits on how devices can behave. First devices need to consider at what power they should transmit. Using too much power puts unnecessary strain on the device's power source. While this does boost the devices signal and thus increases the devices signal to interference ratio, SIR, it also increases interference when other devices try to communicate. This leads to a situation where there are two main reasons for power control. First to optimize power used for transmission and second to increase SIR by decreasing interference {{4 Mehta,Saurabh 2009;}}. It is also clear that all devices transmitting at the maximum power level creates a equilibrium that is not optimal {{44 Goodman,D. 2000;}}. This is usually solved by introducing a free for transmission. By doing so the devices aim to maximize

their SIR while trying to conserve power used for transmission. Studies in power control also seem to focus on providing a certain level quality for all connections {{31 Chi Wan Sung 2003;}}. In the following paragraphs only generalized results are going to be discussed. This is due to the difficulty of mathematics in the articles.

In the paper Goodman and Mandayam formulate a non-cooperative game and formulate means to encourage devices to transmit at a lower power level. This is achieve when a *cost coefficient* is introduced which can be thought to be a fee that devices pay in order to transmit. In the same paper a *net utility function* is former and used in a way that each devise attempts to maximize the payoffs it can receive. The net utility function is a function where the benefits of transmission are divided by the costs of transmission minus a term for transmission, cost coefficient. Because devices aim to maximize their payoffs the results of the net utility function vary from device to device but the end results yield an equilibrium.

{{25 Alpcan,Tansu 2002;}} discuss power control in CDMA uplink. In the paper Alpacan, Basra & Srikant describe CDMA uplink power control as a non-cooperative game and discuss the effects of a pricing strategy to the system. Two different update algorithms are proposed for power control. A Parallel Update Algorithm, PUA, which uses a proposed reaction function to calculate the devices optimal response to the current situation. A second algorithm Random Update Algorithm, RUP, is essentially the same algorithm as PUA with the exception that devices update their power level with a certain probability. Simulations show that in a delay-free system, where all users have the same initial power level, RUA outperforms PUA. In a system with delay, simulations show that PUA performs better than RUA. Two different pricing schemes, a centralized pricing scheme and a decentralized, market-based pricing scheme, are also studied. In centralized scheme devices are divided in to categories and all devices within a certain category have the same SIR requirement. In a decentralized system the base station decides a single price for all devices who choose to pay to in order to achieve a certain level of quality of service. It is shown that "appropriate pricing strategy guarantees meeting the minimum desired SIR level".

{{31 Chi Wan Sung 2003;}} discuss power control for multirate CDMA data networks. The goal of the research is to maximize the throughput of the proposed system. This is achieved when a pricing mechanism is implemented. The only needed information for the mechanism are a pricing parameter and the power level of the received signals plus noise. The system achieves a Nash equilibrium with the proposed parameters. It can be show from the optimal results that

high-rate connections should maintain a higher energy per bit than low-rate connections {{31 Chi Wan Sung 2003;}} meaning that high-rate connections pay more to transmit than low-rate connections.

Power control games for cognitive radio networks and the effects of unlicensed users have on licensed users is discussed by {{28 Wei Wang 2007;}}. In the paper power control for cognitive radio networks are modeled as a non-cooperative game and an exponential payoff function is introduced limit the interference caused to other devices. This is due to the fact that negative effects of interference for licensed users increases dramatically if background noise, i.e. transmissions from unlicensed users, increases. {{28 Wei Wang 2007;}} also simulate the proposed system and evaluate the performance by two metrics. Spectrum efficiency, reachable capacity per Hz, and outage probability. It is shown that the proposed algorithm restricts interference to licensed users with an "acceptable cost on the spectrum efficiency" {{28 Wei Wang 2007;}}. The proposed system also decreases the probability of outages.

It is clear that by analyzing power control with non-cooperative game theory the devices can adjust how much power they should use to reach, or to maintain, a certain level of SIR. This can be achieved by introducing a pricing mechanism that applies to all devices. By doing so the devices try to maximize their SIR while trying to minimize the fees to the pricing mechanism. According to {{17 Charilas,Dimitris E. ;}} game theoretical tools for the analyzation of power control dilemmas can also be used in Orthogonal Frequency Division Multiple Access (OFDMA) networks as well.

1.6.2 Spectrum allocation

According to {{17 Charilas,Dimitris E. ;}} spectrum allocation, or sharing, can be used when trying to find out how to share a limited spectrum with multiple wireless devices. The best outcome would be if the spectrum would be utilized as much as possible. While this situation can also be modeled as a cooperative game as it is done by {{12 Suris,J.E. 2007;}}, in this thesis the situation is going to be analyzed as a non-cooperative game.

Multiple overlapping IEEE 802.22 networks are discussed by {{46 Sengupta,S. 2008;}}. 802.22 networks are wireless regional area networks, WRAN, that are based on cognitive radios. They can perform spectrum sensing and move to another spectrum if a unused spectrum is found. If the used spectrum is accessed by a licensed devices the unlicensed devices must move to another spectrum. In an area with many licensed devices unused spectrum is a commodity that needs to be utilized carefully. As interference between networks increase the

throughput and QoS will be compromised. This is why interference should be minimized. The game proposed game can be solved with a pure or a mixed strategy and a dominant best response strategy is presented. From the results it is clear that when there are few networks competing for the available spectrum, networks are better off switching with a higher probability than in a situation where there are more networks competing for the same amount of available spectrum. It is also shown that if the number networks and available spectrum is increased, the mixed strategy solution always outperforms the pure strategy solution while keeping the costs for convergence. The pure strategy solution are shown to have exponential costs for convergence.

{{47 Gardellin,V. 2010;}} model IEEE802.22 networks just as {{46 Sengupta,S. 2008;}} did with a few exceptions. First, the game is considered to be a multi-player non-cooperative repeated game. Secondly, a different interference model is assumed. Two different utility functions, one to maximize spatial reuse of the spectrum and a second one to minimize interference, are proposed. The proposed algorithms are compared with the results of {{46 Sengupta,S. 2008;}} but due to the differences in interference models that were used straight comparisons in terms of channels used and local/global interference is not possible. However, When the comparing convergence costs the proposed algorithms perform better than the ones proposed in {{46 Sengupta,S. 2008;}}.

A non-cooperative game amongst secondary users in a cognitive radio networks is discussed and analyzed with few different metrics in {{48 Malanchini,I. 2009;}}. In the paper a Spectrum Selection Game is introduced and its behavior with two different types of cost functions are analyzed with simulations. Later a fee for switching channels is introduced and the system is modeled as a repeated game. The future games can be modeled as a subgame to the currently played game. Due to the number of possible future games and unavailability of information about future games {{48 Malanchini,I. 2009;}} decide that solving them is not practical for their case. From the results it can be shown that the social cost, that is the sum of all the costs for all of the users, of the proposed payoff functions have the same results as if the users had cooperated to achieve an optimal solution. Another result is that secondary users prefer spectrum that is rarely used. When a switching fee is introduced the probability for users to switch channels decreases as the free increases. This leads to secondary users not switching to better spectrum slot even if there is one available because the costs of switching out weight the possible gains.

As shown analyzing spectrum allocation in cognitive networks as a non-cooperative game is quite useful and for a more thorough overview on how game theory can be applied in cognitive networks can be found in {{4 Mehta,Saurabh

2009;}} or {{10 Maharjan,Sabita 2010;}}. Fortunately, other networks can also be analyzed with the tools that game theory provides. For example, {{21 Niyato,D. 2008;}} analyze radio resource management in 4G networks and provide a framework for radio resource management which provides a fair resource allocation. Other usages are for example when non-cooperative service providers try to maximize their payoffs by selling extra resources to other service provides{{49 Bennis,M. 2008;}}.

1.6.3 Forwarding and Forwarder's dilemma

Summary

summary about what you just red, where this findings can be applied and future research.

References