Jouni Laitinen

GAME THEORY IN WIRELESS NETWORKS

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ABSTRACT

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1. Introduction

Some general information about why you should read this thesis, where you could find game theory

Game Theory

Merriam-Webster defines game theory, as "the analysis of a situation involving conflicting interests (as in business or military strategy) in terms of gains and losses among opposing players". What this means is that game theory is used to describe decision making situations and the interaction that happens. For this, the mathematical tools that game theory provides are a valuable asset. Due to the fact that the tools that game theory provides are quite abstract, they can be used in various situations ranging from the strictly mathematical to social sciences and information technology. In recent years there has even been some advances in to using game theory to analyze human decision making in a field called Neuroeconomics (Glimcher, 2003; Camerer, Loewenstein and Prelec, 2005). The next chapter will give a more detailed description in to the history of Game Theory.

1.1 History

Game theory has a very long history and the first results of game theoretic modeling can be seen in the Talmud(Aumann & Maschler, 1985) where the results of bankruptcy are considered. Augustin Cournot (1801 – 1877) formed a model of oligopoly(Cournot, 1838) which models the interaction between a small number of sellers. Currently, the field of game theory is considered to have gotten formed in 1944 by the publication of "Theory of games and economic behavior" by John von Neumann and Oskar Morgenster. The book has essential tools to solve games such as backwards induction. As for games the book features zero-sum games, non-zero sum games as well as games with perfect and imperfect information. Most of these are covered later in this thesis.

Next big step froward was when developed by John Forbes Nash. Nash's doctoral thesis included the definition for the Nash Equilibrium. The thesis served as a foundation for four articles that include non-cooperative games(Nash, 1951) and bargaining (1950). For these developments Nash was awarded the Nobel Memorial Price in Economic Sciences in 1994. The price was awarded together with Reinhard Selten and John Harsanyi. All together the Nobel Price has been awarded to eight game theorists. In in recent years game theory has been used as a base of mechanism design (Fudenberg & Tirole, 1991). This field is sometimes called reverse game theory.

1.2 What is a strategic game?

In a game strategic there are different actors who have to make decisions on how to act upon certain rules and have certain preferences over out comes. This means for example that a situation when two friends are deciding on where to eat and they both have a favorite restaurant can be modeled as a strategic game. The definition of such a strategic game is as follows (Osborne, 2004)

- a set of players
- each player has a set of actions
- each player has some preferences over the possible outcomes

In this the players are the two friends. Both of them can choose where they will eat but would prefer to go each own favorite restaurant. The game derived from these premises is usually called the "battle of the sexes" or "Bach or Stravisky".

- Players: the two friends
- Set of actions: each player has the choice between {restaunt1, restaurant2}
- Outcomes: each one prefers to eat at her favorite restaurant

We will analyze this game later.

1.3 Different types of games

There are many ways one could categorize different types of games. Avinash Dixit and Susan Skeath gave a very good list of different types of games in their

book “Games of Strategy”. In the book they divide games in to six different categories(Dixi & Skeath, 2004, 20-27)

– Do the players take turns? If they do the game is called a sequential game and the players make their decision simultaneously the game is called a simultaneous games.
– Zero sum games, that is are the players interests opposite or do they share some interests.
– Repeated games. Is the game going to be played more than once. If played only once then the game is called a one-shot game. If played more than once then the game is called a repeated game.
– Games of perfect information. Do the players know everything about their opponents or is there uncertainty in the game? If there is uncertainty in the game then it is called a game of imperfect information. If one player know more about the game than her opponent(s) then the game is called a game with incomplete, or asymmetric, information.
– Can the rules be manipulated? If the rules can be manipulated then the pregame becomes the real game as this will surely effect the outcome of the game.
– Can cooperation agreements be enforced? If agreements indeed can be enforced then the game is called a cooperative game. If not then the game is called a non-cooperative game.

The games that are going to be looked at can be divided in to two main categories: Cooperative games and non-cooperative games. The reason for this is quite simple because the reasoning and logic behind cooperative game is different than the reasoning behind non-cooperative. In cooperative games the players are trying to maximize their outcomes by working as a group in an environment where agreements can be enforced. This is quite the opposite in non-cooperative games where agreements can not be enforced and therefore it is in every players interest to maximize her payoff. One other reason for dividing them in to these two categories is that both cooperative and non-cooperative games can have games with the other properties listed above within the games themselves. This is to say that there can be a repeated non-cooperative game with imperfect information. That is also going to be one of the examples that is going to be discussed in the section of non-cooperative games in wireless networks.

1.3.1 Cooperative Games

Games of cooperation are games where agreements enforceable (Dixit & Skeath 2004). This means that the players decisions are made in a group and that all

members of the group will act according to the decision or a game where all players act according the agreements that can be forced collectively or directly. By doing so the players cooperate to maximize their payoffs. Working together they can for example form coalitions that aim to increase the payoff of the coalition and thus increase the payoff to each member.

1.3.2 Non-cooperative Games

Non-cooperative games are games where agreements can not be forced and thus the main focus of each player is to maximize their own payoffs. While this does make it seem like every player always acts selfish it is not always the case. In a later chapter there will be an example of a game where cooperation still guarantees the best outcome. Probably the most famous non-cooperative game is called the prisoners dilemma. The ideas behind prisoners dilemma is quite simple. Two friends have committed a crime and been detained by the police. The police have enough information on the two to convict them because of a smaller crime that two did earlier. They also know that the two did a larger crime but they lack the information to prosecute. The police make an offer where the person who rats out the accomplice gets to walk free but the other person gets a longer sentence. If both of them talk then both of them will serve a long sentence. Both prefer walking free to small sentence to a long sentence. This is also how her partner in crime thinks. Therefore, this situation can be formed as the following game

> Payers: the two criminals.
> Strategies: {talk, don't talk}
> Preferences: walk free > short sentence > medium sentence > long sentence

The next chapter provides everything needed in order to analyze and solve this situation.

1.4 How are games solved?

In this chapter covers how different types of games can be analyzed and solved. Due to the fact that this chapter is going to be notation heavy there will be quite a few examples. For more advanced readers should look at (Osborne, 2004) as it contains more formal definitions of the following games. For a reader wanting more explanations (Dixit & Skeath, 2004) might be good because it

contains more thorough explanations and doesn't have as much formal equations as former.

Game theory assumes that the players are rational and acts rationally to maximize their payoffs (Osborne, 2004). What this means is that the player can assign some kind of values to different outcomes of the games and act rationally to maximize it. Players are assumed to have some common knowledge. For more thorough commentary on what rationality implies, (Osborne, 2004, 6-7; Dixit & Skeath, 2004, 29 - 32) is a good place to start. For criticism on the rationality assumption readers can see for example (Osborne, 2004) and (Dixit & Skeath, 2004).

1.4.1 General information about games

Most of the definitions in this chapter are from (Osborne, 2004). Let's assume that the payers playing the game are rational and thus want to maximize the payoff they can achieve. These are called *payoff functions* and they can be represented by a numerical value. For a player to act rationally between two choices a and b, the player prefers a if and only if the payoff function of a is greater than the payoff function of b. I'll use u to represent players preference over the possible outcomes. Thus we have

$$u(a)>u(b)$$

If the two outcomes have the same payoff, the player is indifferent between then and chooses both with the same probability.

1.4.2 Normal form games

The definition of a strategic games in chapter 1.2 was the combination of three things. Players, actions and preferences over the possible outcomes. The game formed in chapter 1.3.2 where two criminals were under investigation. Both preferred going free to short sentence to medium sentence to long sentence Both players decide their actions simultaneously. This game is known as The Prisoners Dilemma and it was developed by Albert Tucker (Poundstone, 1992). There are two ways the game can be modeled. In normal form, where the players outcomes would be put in to a matrix or in strategic form where the outcomes would be put in as leaves of a decision tree. More detailed differences will be covered later. Now to model this situation as a normal form game. The previously formed games to be like this

Players: the two criminals
Strategies: both can choose between {Talk and Silent}
Preferences: going free > short sentence > medium sentence > long sentence

This somewhat resembles the game formed in 1.2. quite closely. In fact both games can be analyzed in normal form. Before analyzing this game payoff functions need to be added to the the preferences. Now to write the payoff functions so that player 1's choice is the first parameter and player 2's choice is the second parameter. Thus, for player 1

$$u_1(Talk, Silent) > u_1(Silent, Silent) > u_1(Talk, Talk) > u_1(Silent, Talk)$$

and for player 2

$$u_2(Silent, Talk) > u_2(Silent, Silent) > u_2(Talk, Talk) > u_1(Talk, Silent)$$

By choosing the following numerical values for player1

$$u_1(Talk, Silent) = 3, u_1(Silent, Silent) = 2, u_1(Talk, Talk) = 1, u_1(Silent, Talk) = 0$$

and for player 2

$$u_2(Silent, Talk) = 3, u_2(Silent, Silent) = 2, u_2(Talk, Talk) = 1\ u_1(Talk, Silent) = 0$$

the following game can be formed

		Criminal 2: Silent	Criminal 2: Talk
Criminal 1	Silent	2 , 2	0 , 3
	Talk	3 , 0	1 , 1

According to chapter 1.4 all players act rationally and thus want to maximize their payoffs. By comparing the different payoffs it is clear that Talk always yields better payoff (3>2, 1>0). This is called Criminal 1' *best response* to Criminal 2's actions. For Criminal 2's Silent Criminal 1's best response is to choose

Talk. For Criminal 2's action Talk Criminal 1's best response is to choose Talk. Therefore the rational thing to do is to always choose Talk. Playing Silent in any situation always gives an outcome that is suboptimal and can be improved by playing Talk. In this situation talk s*trictly dominates* Silent. In an equilibrium situation no player plays strictly dominated strategies. Because no rational player plays strictly dominated strategies they can be removed from the payoff matrix. From the matrix it is clear that Talk strictly dominates Silent. Therefore, Silent can be remove from both players action sets. This leaves on only one possible outcome (Talk, Talk). This means that if both players play rationally they will always play Talk and the game will always end in (Talk, Talk) with the payoffs of (1,1).

1.4.3 Best responce and Nash Equilibrium

John Forbes Nash formed probably the most important solving tool for games in his doctoral thesis. *Nash equilibrium of strategic games with ordinal preferences* expands on the ideas that we previously discussed and can be formulated as follows(Osbourne, 2004, 23).

> ''...for every player *i* and every action a_i of player i, a^* is at least as good according to player *i* preferences as the action profile (a_i, a^*_{-i}) in which player *i* chooses a_i while every other player *j* chooses a^*_{-i}. Equivalently, for every player *i,* $u_i(a^*) \geqslant u_i(a_i, a^*_{-i})$, for every action a_i of player *i,* where u_i is a payoff function that represents player *i*'s preferences.

This means that if no player can improve their payoffs by chancing to another strategy unilaterally then the strategy played is a Nash equilibrium. From the way that the definition is formed it is clear that there might be more than one Nash Equilibrium in a game. By examining the different strategies in the Prisoners Dilemma it is obvious that (Talk, Talk) is indeed a Nash equilibrium. What about the restaurant game. Does it have any Nash equilibrium?

- Players: the two friends
- Set of actions: each player has the choice between {restaunt1, restaurant2}
- Outcome: each one prefers to eat at her favorite restaurant

To give the outcomes an ordinal preference I have chosen $u_i(\textit{favorite restaurant}) = 2, u_i(\textit{other restaurant}) = 1$ and $u_i(\textit{eat alone}) = 0$. Both of the players have symmetrical preferences we have the following game.

		Player 2	
		Restaurant1	Restaurant2
Player1	Restaurant1	2 , 1	0 , 0
	Restaurant2	0 , 0	1 , 2

Clearly there are no strictly dominated strategies. By marking the best responses with a * for both players we have

		Player 2	
		Restaurant1	Restaurant2
Player1	Restaurant1	2 * , 1 *	0 , 0
	Restaurant2	0 , 0	1 * , 2 *

Now there are two cells in which both players have marked their best responses. Turns out that they both are in fact Nash equilibrium because they are the best responses to each others every strategy. Let $B_i(a_{-i})$ be the best response of player *i* to every other players action a_{-i}. From this the following can be formulated(Osborne, 2004, 36).

> The action profile a^* is a Nash equilibrium of a strategic game with ordinal preferences if and only if every player's action is a best response to the other players' action:
>
> a_i^* is in $B_i(a_{-i}^*)$ for every player *i*

From this it is obvious that by inspecting every cell and marking the best responses for every action for every player *i* it is possible find Nash equilibrium in games where there are no dominant strategies.

1.4.4 Mixed strategies

Let's look at a game where there are exits no Nash equilibrium. Classically called ”Matching pennies”. Two players are playing a game where they decide on whether to heads or tails of a penny. The decisions are made simultaneously.

If the players show the same side player 2 pays player 1 one euro. If the players show different sides then player 1 pays player 2 1 euro. Both prefer to receive money to losing money. To make this in to normal form game

		Player 2	
		Heads	Tails
Player1	Heads	1 , -1	-1 , 1
	Tails	-1 , 1	1 , -1

Clearly there are no dominant strategies and the best responses are placed as follows.

		Player 2	
		Heads	Tails
Player1	Heads	*1 , -1	-1 , *1
	Tails	-1 , *1	*1 , -1

It appears that there are no Nash equilibrium in this game.

Wireless technology

Future technology and what limitations it will have. Talk about cognitive radios and stuff.

Game Theory in wireless networks

Why wireless networks can be analyzed with game theory? It seems that there are no connections between the abstract ideas of von Neumann and Nash and your favorite hand held device.

1.5 Cooperative games in wireless networks

Cooperative games in wireless networks all have one assumption in common. They all need to have some way of enforcing agreements that are made. This can be done by making standards within the industry or by having an outside institution enforce the agreements. The outside institution can be the FCC, for example. Unfortunately, if agreements can not be forced then the game becomes a non-cooperative game. Another threat to these types of networks where nodes cooperate is that if a non-cooperative node participates in the game, it can free ride on the results of others without making any contributions to the network. Therefore, most games that are going to be presented in this thesis are going to be non-cooperative games.

1.5.1 Different types of cooperative games

As stated in the previously cooperative games need a way to enforce agreements and when that requirement is fulfilled there are numerous scenarios that can be modeled. These include, but are not limited to, channel access time, spectrum assignment, channel allocation, packet forwarding and spectrum sensing (Maharjan, Zhang & Gjessing 2010; Charilas, Panagopoulos 2010).

1.6 Non-cooperative Games in wireless networks

Stuff about non-cooperative games in wireless networks. Forwarder's dilemma(cooperation in scarcity), folk theorem and strategies(Always cooperate, always defect, TFT, Stochastic-TFT, Expected utility).

Summary

summary about what you just red, where this findings can be applied and future research.

REEFERENCES

Guess what?

LIITE 1 ENSIMMÄINEN LIITE

Tämä on tutkielman mahdolliselle liitteelle varattu sivu. Mikäli liitteitä on useampia, ne sijoitetaan omille sivuilleen. Eli jos tässä raportointipohjassa olisi liite kaksi, se alkaisi omalta sivultaan.

TAULUKKO 1 Esimerkki taulukon selitetekstistä

Dokumenttityyppi	**Muistio**	**Lasku**
Organisaation logo	Ei esiinny	Esiintyy
Yhteystiedot	Esiintyy	Esiintyy
Laatijan nimi	Esiintyy	Ei esiinny

Seuraavassa esimerkki kuviosta mahdoton kolmio (kuvio 1):

KUVIO 1 Mahdoton kolmio